

RECENT METHODS IN SOLVING MULTICRITERIA DECISION MAKING PROBLEMS

*Thesis
Submitted for the Degree
of Doctor of Philosophy*

**In
Pure Mathematics**

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1995

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وَقُلْ رَبِّ زِدْنِي عِلْمًا

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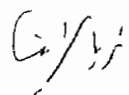
Ph.D. Thesis (Pure Mathematics)

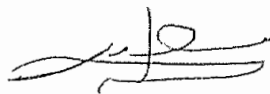
Title of Thesis:

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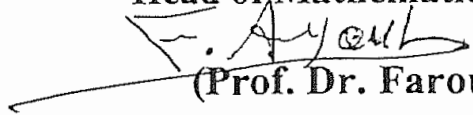
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ACKNOWLEDGEMENT

I would like to express my deepest gratitude and thanks to ***Prof. Dr. M. S. A. Osman***, Professor of Pure Mathematics and Operations Research and Head of Department of Basic sciences higher Technological Institute, Tenth of Ramadan City for suggesting the problems, his continuous help and active discussions throughout the developement of this thesis. His kind supervision has been most helpful at all stages.

My sincere thanks are also due to ***Prof. Dr. Soraya Sherif***, Professor of Pure Mathematics, University College for Women, Ain Shams University for her encouragement and her kind help.

I must acknowledge also ***Dr. Elsayed A. H. Eltelt***, Assistant Prof. of Pure Mathematics MTC, Cairo, Egypt, for revising the manuscript of this thesis, and for helpful suggestions.

Mona Mohamed M.M. Tarrad

ABBREVIATIONS AND NOTATIONS

M-CDMP = Multi -Criterion Decision Making Problem.

VOA = Vector Optimization Approach.

GPA = Goal Programming Approach.

DIA= Decision Interactive Approach.

VOP = Vector Optimization Problem.

GTN = Generalized Tchebycheff Norm.

(PT) = Augmented weighted Tchebycheff Program.

GPP = Goal Programming Problem.

GP = Goal Programming.

MDP= Multiobjective Decision Problem.

DR = Decision Rule

DM = Decision Maker.

SWT =Surrogate Worth Trade - off.

KTCN= Kuhn -Tucker Conditions For Noninferiority.

*KTSCN= Kuhn -Tucker Saddle Point Conditions For
Noninferiority.*

*M-ONLPP = Multi-Objectives Nonlinear Programming
Problems.*

GDF = Dyer, Geoffrion and Feinberg.

MOMD = Multi-Objective Mathematical Programming

DV = Decision Variable.

S = Feasible Set.

OS = Objective Space

DS = Decision Space.

U = Utility Function.

Q = Pairwise Preference Criteria.

QL = Least - Preferred Alternative.

QM = Most - Preferred Alternative

U_{qv} = Quasiconcave Utility Function

U_{qx} = Quasiconvex Utility Function.

AR = Relaxed Assumptions

*(IM-ONLPP)_c = Interactive Approach For Solving Multi-
objective Nonlinear Programming Problems
Combined.*

*(IM-ONLPP)_u = Interactive Approach For Solving
Mukltiobjective Nonlinear Programming Problems
Unified.*

*IGGPP = Interactive Generalized Goal Programming
Problem.*

IEM = Interactive Efficient Method.

*IM-ONLPP = Interactive Multiobjective Nonlinear
Programming Problems.*

LLP = Linear Programming Problem.

NLPP= Nonlinear Programming Problem.

*FVOP = Vector Optimization Problems With Fuzzy
Parameters.*

*FGGPP = Generalized Goal Programming Problems With
Fuzzy Parameters.*

α - VOP= Nonfuzzy Vector Optimization Problem.

*α -GGPP= Non fuzzy Generalized Goal Programming
problem.*

LP= Linear Programming

MOPP= Multiobjective Programming Problem.

CPP= Convex Programming Problem

NLP= Nonlinear Programming

KTSP= Kuhn-Tucker Saddle Point

GGPP= Generalized Goal Programming Problem

ABSTRACT

This work aims to give a comprehensive study in recent methods for solving multicriteria decision making problems (M-CDMP). Also a new method to determine the stability set of the first kind has been introduced for M-CDMP without differentiability.

The author presents two interactive algorithms for solving multi-objective nonlinear programming problems. She makes also a comparative study between them and an interactive algorithm for solving generalized goal programming problem.

The researcher presents two interactive algorithms for solving M-CDMP under fuzzy environment in both vector optimization problems and goal programming problems.

The author presents method for determining the stability set of the first kind for M-ONLPP without differentiability. This method depends on the procedure for solving the problem.

INTRODUCTION

Multiple criteria decision making (M-CDMP) is concerned with methods and procedures by which multiple criteria can be formally incorporated into the analytical process.

The (M-CDMP) elicits in wide variety of problems, such as vector maximization, goal programming, group decision problems (with several criteria), multi - attribute problems, and utility and theory of measurements.



Vector optimization problems (VOP), or multiobjective optimization problems are one main branch of mathematical optimization. (VOP) appear when a decision maker must take a decision satisfying the optimization of more than one conflicting objectives.

Stability analysis in multiobjective nonlinear programming has been extensively investigated from the qualitative point of view. The stability notions are the sets of parameters that retain specific features for the optimal solutions of the multiobjective nonlinear programming problems.

This thesis consists of six chapters for a characterization and further investigation of the solution of multicriteria decision making problems (M-CDMP).

Chapter 1 is a survey on some approaches for treating multicriteria decision making (M-CDMP) problems.

In Chapter 2, the author presents interactive algorithms for solving multiobjective nonlinear programming (M-ONLP) problems. She presents an interactive method for solving general nonlinear multiple objective optimization problems (NLMOOP) to get the best compromise solution in situations

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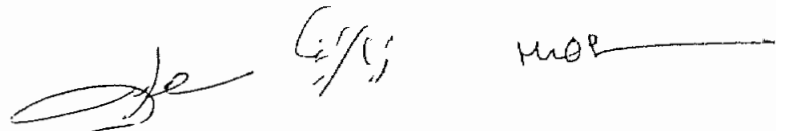
with an implicitly defined utility functions which are quasiconcave. She presents also an interactive method for solving multiobjective nonlinear programming problems (M-ONLPP) she uses a combined Tchebycheff and aspiration criterion vector techniques.

In Chapter 3, the author presents an interactive algorithm for solving Generalized Goal Programming (GGPP) problems by using visual interactive method. And she presents also a comparative study between the algorithms and solve an illustrative example by the three methods respectively.

In Chapter 4, the author presents interactive algorithms for solving (M-CDMP) problems under fuzzy environment in both (VOP) problems and (GPP) problems. By using a combined Tchebycheff and Aspiration criterion methods for VOP and visual interactive methods for GPP.

In Chapter 5, the researcher presents a method for determining the stability set of the first kind for multiobjective nonlinear programming problems without differentiability. She uses several examples with parameters in the objective functions or in the constraint functions to clarify the method.

Chapter 6 contains the conclusions of this thesis and the recommendations for further research in this field.

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CONTENTS

<i>Subject</i>	<i>Page</i>
 Chapter 1: SURVEY ON THE METHODS FOR SOLVING (MCDMP) PROBLEMS.	
1.1: Vector Optimization Approach	1
1.1.1: Fundamental definitions and concepts.	2
1.1.2: Some common approaches for characterizing efficient solutions	2
1.1.3: Kuhn - Tucker conditions for noninferiority (KTCN)	8
1.1.4: Basic stability results for multiobjective convex program.	1 1
1.2: Goal Programming Approach	
1.3: Interactive Multiobjective Programming Methods.	17
1.3.1: Methods based on global preference.	18
1.3.2: Methods based on weights, priorities, goals and ideals.	2 2
1.3.3: Methods based on trade-offs.	2 6
 Chapter 2: INTERACTIVE APPROACHES FOR SOLVING MULTIOBJECTIVE NONLINEAR PROGRAMMING PROBLEMS.	
2.1: The First Interactive Approach For Solving Multiobjective Nonlinear Programming Problems Unified (IM-ONLPP) _U .	2 9
2.1.1: Problem formulation and description of the methods.	30
2.1.2: Algorithmic steps of the interactive efficient method (IEM) with quasiconcave utility function.	4 1

2.2: The Second Interactive Approach For Solving Multiobjective Nonlinear Programming Problems Combined (IM-ONLPP) _c .	50
2.2.1: Problem formulation.	51
2.2.2: Interactive efficient reduction method.	53

Chapter 3: A COMPARATIVE STUDY ON SOME INTERACTIVE APPROACHES FOR SOLVING (MCDM) PROBLEMS.

3.1: An Interactive Approach For Solving Generalized Goal Programming Problems (IGGPP).	66
3.1.1: Problem formulation.	
3.1.2: An interactive algorithm for solving (GG) problems.	67
3.2: Comparative Study On Interactive Approaches.	69
3.2.1: Problem structure.	73
3.2.2: Solving techniques.	74
3.2.3: Decision rule.	76
3.2.4: Stopping rule.	
3.2.5: Output of the method.	78
3.2.6: The advantages of the method.	
3.2.7: The disadvantages of the method.	
3.2.8: The complexity of the parametric study.	80
3.2.9: Example.	83

Chapter 4: INTERACTIVE APPROACHES FOR SOLVING (MCDM) PROBLEMS UNDER FUZZY ENVIRONMENT.

4.1: An Interactive Approach For Solving Vector Optimization Problems With Fuzzy Parameters.	90
4.1.1: Formulation of (FVOP).	91

4.1.2: Algorithm for FVOP.	94
4.2: An Interactive Approach For Solving Generalized Goal Programming Problems With Fuzzy Parameters.	102
4.2.1: Problem formulation.	104
4.2.2: Interactive algorithm for solving (∞ -GGPP).	113
<i>Chapter 5: A METHOD TO DETERMINE THE STABILITY SET OF THE FIRST KIND FOR NONDIFFERENTIABLE MULTIOBJECTIVE PARAMETRIC NONLINEAR PROGRAMING PROBLEMS</i>	
5.1: Statement Of The Problem :	121
5.1.1: First case: Parameters in the constraints.	
5.1.2: The second case: Parameters in the objectives.	123
5.2 : The Method For Determining The Stability Set Of The First Kind Without Differentiability For (MCDMP) Problems	124
5.3 : First Example	126
5.4 : Second Example	132
5.5: Third Example	136
<i>Chapter 6: RECOMMENDATIONS AND POINTS FOR FURTHER RESEARCH.</i>	
6.1: Conclusions.	141
6.2: Points For Further Research	
Bibliography	144
Arabic Summary	