

Ain Shams University
Faculty of Engineering

**Modeling of Multimode Interference Electro-optical
Switching Structures on a Semiconductor Substrate**

By
Eng./ Hatem El-Sayed Hany El-Refaei

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Supervised By

Prof. Dr. / Abd El-Rahman Hasan El-Derby
Dr./ Salah El-Dien Hasan Gamal El-Dien
Dr./ Diao Abd El-Meguid Mohamed Khalil

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Abstract

A new rigorous modal analysis is proposed for analyzing multimode interference (MMI) structures. This technique is based on describing the optical field in the input and output waveguides by their guided and radiation modes while in the multimode section by its guided modes only since usually it supports large number of them. We account for the reflection occurring at both input and output interfaces by considering the continuity of the electric and magnetic fields. Hence we could define a transmission and reflection matrices for each interface. The formulation of the propagator in matrix form enables us to account for the multiple reflection inside the structure. We finally achieve an analytical formula for transmission and reflection matrices that relate the input modes' amplitudes to the transmitted and reflected modes' amplitudes taking multiple reflections into account. This technique is adequate for both strong and weak guiding structures. It shows results that resemble experimental published data. It shows also good results in comparison with Beam Propagation Method (BPM) for weak guiding structures and Self Imaging Technique (SIT) for strong guiding structures. This technique could also predict short distances for the multimode section for new self images using weak guiding structure that are verified by the BPM and ray optics.

List of Symbols

| | |
|---------------|---|
| \bar{a} | Polarization unit vector. |
| $A(r,t)$ | Potential vector of electromagnetic wave. |
| $A(\sigma)$ | Radiation mode amplitude. |
| α | Absorption coefficient. |
| α_p | Index loss variation ratio. |
| β | Mode propagation constant in the z direction. |
| c | Free space speed of light. |
| c_v | Amplitude of guided mode. |
| $N_V^{nq}(z)$ | Hole envelope function. |
| $N_C^p(z)$ | Electron envelope function. |
| dn | Refractive index difference. |
| $\delta(z)$ | Delta-Dirac function. |
| Δn | Induced change in the real part of the refractive index. |
| $\Delta n''$ | Induced change in the imaginary part of the refractive index. |
| E_c | Conduction band edge. |
| E_f | Fermi level energy. |
| E_g | Band gap energy. |
| $E_{initial}$ | Initial electron energy. |
| E_{final} | Final electron energy. |
| E_v | Valence band edge. |
| $E(x,z)$ | Electric field distribution. |
| ϵ_0 | Free space permittivity. |

| | |
|------------------------|---|
| ϵ_r | Relative permittivity. |
| $f(E)$ | Fermi-Dirac distribution function. |
| $\phi_g(x, \beta_g)$ | Field distribution of guided modes. |
| $\phi(x, \sigma)$ | Field distribution of radiation mode. |
| γ_c | Cross talk at cross state. |
| γ_d | Cross talk at direct state. |
| Γ | Extinction ratio. |
| \hbar | Plank's constant divided by 2π . |
| $H(x, z)$ | Magnetic field distribution. |
| H'_{cv} | Matrix element. |
| H'_{op} | Perturbation operator. |
| I_{av} | Average field intensity. |
| I_o | Input field intensity. |
| \bar{I} | Identity matrix. |
| K | Boltzmann constant. |
| \bar{k}_c, \bar{k}_v | Electron momentum vector in conduction and valence bands. |
| k_x, k_y | Mode propagation constant in the x, y directions. |
| \bar{k}_{ph} | Photon momentum vector. |
| κ | Coupling constant. |
| L_π | Beating length. |
| λ_o | Free space wavelength. |

| | |
|------------------------|---|
| m | Free electron mass. |
| m_e^* | Electron effective mass. |
| m_h^* | Hole effective mass. |
| m_r^* | Reduced effective mass. |
| μ_0 | Free space permeability. |
| $n(x,y,z)$ | Refractive index distribution. |
| n', n'' | Real and imaginary parts of the refractive index. |
| n_c | Cladding refractive index. |
| n_{eff} | Mode effective index. |
| n_g | Guide refractive index. |
| N_j^{3D} | Three dimensional joint density of states. |
| \bar{P}_{cv}^{2D} | Two dimensional momentum matrix elements. |
| \bar{P}_{cv}^{3D} | Three dimensional momentum matrix element. |
| q | Electron charge. |
| \vec{r} | Position vector. |
| R | Absorption rate. |
| \bar{R} | Reflection matrix. |
| R_{ex} | Exciton Rydberg energy. |
| $\bar{R}_{mult.-ref.}$ | Reflection matrix with multiple reflection. |
| ρ_{2D} | Two dimensional density of states. |
| ρ_{3D} | Three dimensional density of states. |

| | |
|-------------------------------|--|
| S | Spin quantum number. |
| $S(k_1, k_2)$ | Transition probability. |
| σ | Radiation mode transverse propagation constant. |
| T | Temperature in Kelvin degree. |
| \bar{T} | Transmission matrix. |
| $\bar{T}_{mult.-ref.}$ | Transmission matrix with multiple reflection. |
| $U_v(\bar{r})$ | Electron periodic part of the initial wave function. |
| $U_c(\bar{r})$ | Electron periodic part of the final wave function. |
| V | Applied voltage. |
| W | Quantum well width. |
| W_{eff} | Guide effective width. |
| W_{ph} | Guide physical width. |
| ω | Angular frequency. |
| $\psi_{k_1}(t, \bar{r}, k_1)$ | Initial electron wave function. |
| $\psi_{k_2}(t, \bar{r}, k_2)$ | Final electron wave function. |
| $\psi_v(x)$ | Field distribution of guided mode. |
| $\psi(x, y, z)$ | Electric field component of electromagnetic wave. |
| $\Psi(x, z)$ | Any field distribution. |

Key Words

| | |
|--------|---|
| BPM | Beam Propagation Method. |
| EIM | Effective Index Method. |
| FFTBPM | Fast Fourier Transform Beam Propagation Method. |
| FI | Fourier Image. |
| MMI | Multimode Interference. |
| MQW | Multiple Quantum Well. |
| QW | Quantum Well. |
| RSM | Radiation Spectrum Method. |
| SC | Semiconductor. |
| SI | Self Imaging. |
| SIT | Self Image Technique. |
| TE | Transverse Electric. |
| TM | Transverse Magnetic. |
| VLP | Voltage Length Product. |

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List of Figure Captions

Fig.(1.1)

Schematic cross section view of famous two dimensional structure (a) embedded , (b) ridge , (c) inverted ridge, (d) strip loaded, (e) rib, and (f) diffused waveguides.

Fig(1.2)

Y-junction.

Fig.(1.3)

Shaped Y-branch

Fig.(1.4)

Switching curves for shaped Y-junction switch where $\theta_1=0.5^\circ$, $\theta_2=0.2^\circ$, Guide width = $2.3\mu\text{m}$, branch length = $550\mu\text{m}$, and total device length = 1.3mm operating at (a) $\lambda = 1.52\mu\text{m}$, and (b) $\lambda = 1.58\mu\text{m}$

Fig(1.5)

Conventional directional coupler.

Fig.(1.6)

Directional coupler switch

Fig.(1.7)

Intersectional waveguide switch

Fig.(1.8)

(a) Direct state under no applied field and (b) cross state under biasing potential.

Fig.(1.9)

Two mode interference coupler.

Fig.(1.10)

2×2 Multimode interference coupler.

Fig. (1.11)

Schematic representation of (a) direct and (b) cross states of general 2×2 optical switch.

Fig. (1.12)

Intersectional waveguide structure for an optical switch.

Fig. (1.13)

Normally bar state intersectional optical switch using negative refractive index variation[24].

Fig.(1.14)

Normally cross state intersectional optical switch using positive refractive index variation [24].

Fig.(1.15)

(a) 3-D view and cross-sectional view of GaInAs/InP MQW intersectional optical switch. (b) Schematic top view with waveguide pattern dimension [26].

Fig.(1.16)

Strip loaded structure with dashed lines at interfaces where boundary conditions must be fulfilled

Fig.(1.17)

(a) General inhomogeneous waveguide medium and (b) the equivalent lens waveguide.

Fig. (1.18)

(a) Vector representation of propagation constants in different directions, (b) field at different planes in one cell of a lens waveguide. One cell of a lens waveguide consists of a straight homogeneous medium and thin lens.

Fig. (2.1)

(a) Photon absorption by an electron in the valence band, and (b) stimulation emission of a photon by an electron in the conduction band and an incident photon.

Fig.(2.2)

Approximated parabolic function for direct band gap SC material.

Fig.(2.3)

Absorption coefficient due to direct band to band transition in bulk SC material.

Fig.(2.4)

Absorption coefficient due to direct band to band transition in bulk SC with excitonic effect.

Fig.(2.5)

Absorption coefficient of the bulk GaAs measured near the band edge at different temperatures.