ON RECENT APPROACHES FOR TREATING VECTOR OPTIMIZATION PROBLEMS

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By

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INTRODUCTION

Decision with multiple objectives are quite relevant in government, military, industry and other organizations. Researchers from various organisations such as mathematicians, managers, scientists, economists, engineers and others, have contributed to the solution methods for multiple objective optimization problems (MOP).

Multiple-objective optimization problems or vector optimization problems are concerned with determining scalar values for N continuous decision variables in order to maximize (or minimize) n ($n \geq 2$) objectives simultaneously. In order to make the problem nontrival, it is assumed that the objectives are in conflict and incommensurable.

Due to the conflicting nature of the objectives, an optimal solution that simultaneously maximizes (or minimizes) all the criteria is usually not obtainable. Instead there are several solutions, called efficient solutions, that have the property, that no improvemment in any one objective is possible without sacrificing on one or more of the other objectives.

In the last two decades, most research has been concerned with developing solution methods. Most of these solution methods require input from the decision-maker (DM), usually, in the form of preferences. These preferences can be articulated before the mathematical solution process (a prior articulation of preferences), during the solution process (progressive articulation of preferences), after the solution process (a posterior articulation of preferences).

In the first approach, the DM's preference function is assessed or the DM's aspirations are disclosed, even before the MOP is solved. Thus, the MOP is reduced to either a single objective optimization problem (Utility theory), or a series of scalar optimization problems (lexicographic or goal programming). However, determination of the explicit form of the preference function, as required in utility theory, may require a prohibitive level of time and effort. Even though goal programming requires only a pairwise comparison of objectives, it assumes a very simple structure of the preference function that does not allow tradeoffs between objectives.

The second approach does not require a priori preference information, instead it elicits the DM's preference structure through man-machine interactions. Approaches in this category may also be classified as posterior articulation or progressive articulation, depending on the timing of articulation. Posterior articulation suffers from difficulties in generating all efficient solutions. In progressive articulation, also called the interactive methods, at every iteration the DM provides preference information about the current solution either implicity or explicitly. Since the DM is involved in the entire solution process.

Examples of methods in this category are convergent cutting plane algorithm [26]. The STEM method [7], the sequential proxy optimization technique [40], complex search algorithm [22], the direction searching algorithm [30] and interactive efficient tradeoff cutting plane method [43]. In this work, we modify the cutting plane algorithm by using two different interactive methods of calculating weights to preserve the interacting procedure of the approach and its computer code. Also, we give a compartive study of these methods and construct a unified algorithm which combines all these methods.

The third approach, attention is focused on the generation of all efficient solutions, and the set of efficient solutions is either partially or completely enumerated and presented to the DM.

There are several possible forms of scalarizing VOP., see [46],[38],[19],[21]. This research is devoted to deduce a general unified approach. In will be deduced from solving all types of VOP. Our unified approach generilises most of the available known approaches. Also, we construct a new algorithm for finding the set of efficient solutions of VOP.

The main object of this thesis is the study on recent approaches for treating vector optimization problems (VOP)

This thesis consists of six chapters:

Chapter I is concerned with a survey on the most important theorems, definitions and methods for solving VOP problems.

Chapter II is devoted to the comparative studies on interactive approaches for solving VOP.

Chapter III is divided into two sections. The first section introduces a modified interactive cutting-plane algorithm and its computer code is suggested In the second section, a unified interactive approach is presented.

Chapter IV is divided into two sections, In the first section a unified approach for treating vector optimization problems is proposed. In the second section, an algorithm for finding the set of efficient solutions for VOP is suggested.

Chapter V is divided into three sections. In the first section a unified approach for solving VOP with parameters in both the objective function and the constraints is presented. The second section introduces a modified interactive approach for solving MDMP with parametric study.

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In the third section, a unified interactive approach for solving MDMP with parameters in the objective function is suggested.

Chapter VI Contains the conclusions and the recommendations for future work.

CHAPTER 1
SURVEY ON THE THEORY AND METHODS FOR SOLVING VECTOR OPTIMIZATION PROBLEMS (VOP)

In this chapter a survey on previous work concerning the vector optimization problem (VOP). or multiobjective optimization problem (MOP) is introduced. This survey includes theorems,definitions and the different combined approaches for solving vector optimization problems (VOP). Also, a survey on parametric nonlinear programming problems, including three main approaches and formulations of goal programming problems are presented.

1.1 Mathematical Formulation of (VOP)

The (VOP) can be formuluted as

(VOP): $\min\ (f_1(x),\ \ldots,\ f_n(x))$

subject to

$x \in X = \{\ x \in R^N \mid g_i(x) \leq 0\ ,\ i = 1,2,\ldots,\ m\ \}$

where $f_j(x)$, $j=1,2,\ldots,n$ denotes the real valued function that represents the objective punction or decision criteria, and X is the feasible region of the system and $g_i(x)$, $i=1,2,\ldots m$ denotes tthe real valued function that represents the constraints.

1.2 Basic Definitions:

Definition 1

x^* is said to be an efficient solution of VOP if there exists no other $x \in X$ such that $f_j(x) \leq f_j(x^*)$ for all $j=1,2,\ldots,n$ with strict inequality for at least one j.

Definition 2

An efficient solution of VOP X^* is said to be proper efficient solution of VOP if there exists a scalar $k > 0$ such that for each i, $i=1,2,\ldots,n$ and each $x \in X$ satisfying $f_i(x) < f_i(x^*)$, there exists at least one $i \neq j$ with $f_j(x) > f_j(x^*)$ and $(f_i(x) - f_j(x^*)) \leq k\,(f_j(x^*) - f_i(x))$

Definition 3

Stability of multiobjective convex programming (MOCP) problem.

Let $\psi_j(x) = \inf \{f_j(x) \mid G(x) \leq \gamma\}$, $j=1,2,,\ldots,n$, $\gamma \in R^m$

and $G(x) = [g_1(x),,\ldots, g_m(x)]^T$, then the problem VOP is said to be stable if $\psi_j(o)$ are finite and there exist scalars L_j such that

$$[\psi_j(o) - \psi_j(y)] / ||y|| \leq L_j \quad \text{for all } \gamma \neq o , \quad j=1,2,\ldots,n$$

wher $||\ \ ||$ is any norm of interest.