ANALYTICAL METHODS OF CALCULATING THE LIFETIME DISTRIBUTION OF THE RELIABILITY FUNCTION OF REDUNDANT SYSTEMS

THESIS

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By

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بسم الله الرحمن الرحيم

PREFACE

The thesis deals with analytical methods for calculating the lifetime distribution of the reliability function of redundant systems. It consists of seven chapters. The zero chapter exposes some basic concepts of reliability theory, while the material of the next six chapters is supposed to be new in the subject.

In chapter zero we describe some characteristics of reliability. The lifetime, the failure rate and the availability coefficient for reliable systems are discussed. Also the reliability and the mean lifetime for some redundant systems are considered.

Chapter I deals with an analytical method for calculating the lifetime distribution for six models of standby redundant systems (loaded, non-loaded and lightly loaded systems with and without repair). Also the effect of repair and of the choice of its distribution on the mean lifetime are discussed and a numerical example is given. The results in [2,9] can be obtained as special cases of our results in this chapter. The material of this chapter has been published in the Proceedings of the Fifth Annual Operations Research Conference, Zagazig University and The Egyptian Society for Operations Research Application, Vol. 5, No. 1, December 1978.

Chapter II studies the reliability functions and the mean lifetimes for a duplication redundant system with two models of preventive maintenance and repair. Explicit expressions for Laplace transforms of the mean down times of the systems during the period (0,t) and for the mean times to system failure have been obtained under the assumption that the repair and the preventive maintenance times of a unit are differently and arbitrarily distributed.

Moreover the mean lifetimes for some systems with repair and preventive maintenance and with repair only which had been studied before in [10,26,27] are derived as special cases of the results ~~of this~~ chapter. ~~The~~ material of this chapter is mostly new.

Chapter III is concerned with determining the reliability function and the availability of a duplication redundant system with a single service facility for preventive maintenance and repair. The repair times and the preventive maintenance times of the two units are governed by distinct arbitrary general distributions. The results obtained by Srinivasan and Gopalan [35] and by Gopalan and Souza [14] are derived as special cases of our results in this chapter.

Chapter IV is concerned with a renewable redundant system with arbitrary distribution for the volumes of the inputs. The transition probabilities of the states of the system are derived and an expression for the stationary distribution of the number of units giving services and those in repair is given. Also, the probability losses for the stationary state of the system is found. The Erlang's loss formula [12] and the results obtained in [21] are derived as special cases of the results of this chapter. The material of this chapter has been accepted for publication in "The Proceedings of the Egyptian Academy of Sciences".

Chapter V deals with an analytical method for calculating the availability coefficient for a renewable control system consisting of an automatic controller and a controlled unit. The availability coefficient of the system is obtained and in order to make this coefficient available for application an approximate formula is derived. Moreover a numerical example is given to show that the values of the availability coefficient and the corresponding approximate values are very close. The material of this chapter has been accepted for publication in "The Egyptian Computer Journal", The Institute of Statistical Studies and Research, Cairo University.

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In chapter VI an analytical method is used for determining the failure rate for a renewable periodical controlled system. We consider an unrestricted renewable system functioning under a continuous controllable regime. We investigate its assymptotic behaviour and the availability function for the stationary regime. Moreover the failure rates of some special distributions for the lifetime are derived.

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CHAPTER O

SOME CHARACTERISTICS OF RELIABILITY

In the pursuit of maximum efficiency in the operation of large-scale military and commercial engineering systems, the complexity of equipments and configurations has been an ever-increasing phenomenon. Reliability theoryis composed of the systematic procedures based on analytical techniques employed to ensure the operational efficiency of such complex systems. As a quantitative measure, reliability is defined as the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions. Redundance, renewal and maintenance are considered in order to maximize the efficiency of the operation of systems. The mean lifetime and the availability coefficient are considered as quantitative measures for the efficiency of systems.

0.1. BASIC CONCEPTS OF RELIABILITY THEORY

Just as in other branches of science, the basic concepts of reliability theory are understood by describing the relationships among them. By a unit is meant an element, a system, a part of a system or the like. The operation of a unit means the set of all phases of its existence: transportation, maintenance, preparation for a specified use, servicing and repair. The concept of the reliability of a unit is connected in a very real way with the concept of its quality. The quality of a unit is the set of properties defining the degree of suitability of the unit for a specified use. Thus, the concept of the quality of a unit depends in a very real way on the manner in which it is used.

By the reliability of a unit we mean the ability of the unit to maintain its quality under specified conditions of use. In other words, reliability is a property which is extended in time. Reliability is determined by quality and operating conditions.

One of the basic concepts in reliability theory is that of failure and failure-free operation. Failure-free operation is the ability of the unit to keep its ability to function (i.e. not to have failures) throughout a specified period of time under specified conditions. A failure is the partial or total loss or modification of those properties of the units in such a way that their functioning is seriously impeded or completely stopped.

Now, we shall consider the operation of a unit until first failure and we shall study the different characteristics of the reliability of such units. Suppose that the unit begins to function at the instant $t = 0$ and that a failure occurs at the instant $t = \tau$. We say that τ is the lifetime of the unit,

τ being a non-negative random variable with probability density function f(t), zero for negative values of t, that is

$$f(t) = \lim_{\Delta t \to 0+} \frac{\text{Prob}(t < \tau \leq t + \Delta t)}{\Delta t} \qquad (0.1.1)$$

with

$$\int_0^\infty f(t)dt = 1 \quad . \qquad (0.1.2)$$

The distribution of τ is determined by the probability density function, f(t), but it is for some purposes convenient to work with other functions equivalent to f(t). One such is the cumulative distribution function, F(t), giving the probability that a component has failed by time t. That is

$$F(t) = \text{Prob}\ (\tau \leq t)$$
$$= \int_0^t f(u)du. \qquad (0.1.3)$$

F(t) is known as the failure function. Clearly F(t) is a non-decreasing function of t with F(0)=0, F(∞)=1. Equation (0.1.3) gives F(t) in terms of the probability density function, f(t). Conversely, on differentiating (0.1.3),

$$f(t) = F'(t) = \frac{d}{dt}\ F(t) \qquad (0.1.4)$$

determining f(t) for a given F(t).

For some purposes it is slightly more convenient to work with the function complementary to F(t). This is known by the reliability function or the survival function, R(t),

$$R(t) = \text{Prob}\ (\tau > t)$$

$$= 1 - F(t)$$

$$= \int_{t}^{\infty} f(u)du \quad , \qquad (0.1.5)$$

giving the probability that a component has not failed up to time t. Clearly $R(0)=1$, $R(\infty)=0$ and $R(t)$ is a non-increasing function of t. Also

$$f(t) = -R'(t) \quad . \qquad (0.1.6)$$

Figure 0.1 shows the functions $f(t)$, $F(t)$ and $R(t)$ for a typical distribution. Both functions $F(t)$ and $R(t)$ often arise very naturally, for example in considering the probability that a component will survive a guarantee period t_o.

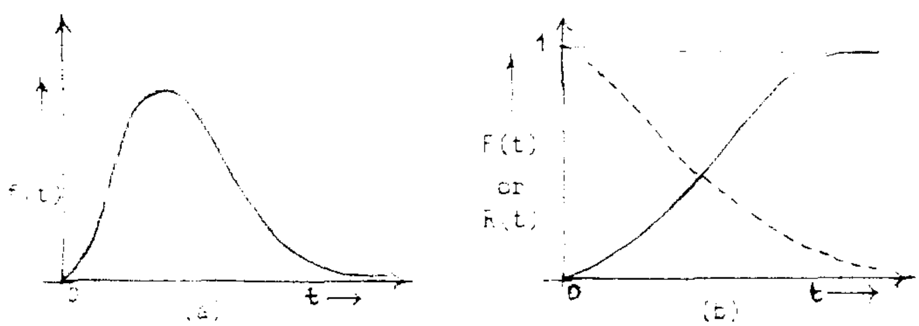

Fig.0.1 Functions $f(t)$, $F(t)$ and $R(t)$ for a typical distribution.
(a) Probability density function, $f(t)$.
(b) ——— cumulative distribution function, $F(t)$
------ reliability function, $R(t)$.

The reliability function $R(t)$ can be found approximately from experiment. Let us suppose first that we need to find the value of this function at $t=t_o$, i.e., the probability of failure-free operation during the interval $[0, t_o]$. We test N identical units under identical conditions during the interval $[0, t_o]$. Suppose $n \leqslant N$ units continued operating without failure up to the instant at which the tests are discontinued. We may consider our experiment as a sequence of N independent trials, in each of which, one or the other of two events takes place: either the unit fails or it does not fail. But then, the ratio n/N is the frequency of occurrence of the first event and, in accordance with Borel's theorem [9], we have

$$n/N \longrightarrow R(t_o) \qquad \text{as} \quad N \longrightarrow \infty$$

with probability 1. This means in practice that, for large N, the approximation

$$n/N \approx R(t_o) \tag{0.1.7}$$

holds with probability close to 1.

If we wish to find the function R(t) for all values of $t \leqslant t_c$, we need to carry out tests over a period of time t_c and to note the instants at which failures occur. If we know these instants, it is easy to determine the function n(t), which is equal to the number of units that have not failed up to the instant t. At the initial instant, this function is equal to n(o)=N, and at the instant of each failure, it decreases by unity. The ratio $R_N(t)=n(t)/N$ is called the empirical reliability function. With increasing N, this function approximates the function R(t) uniformly, and, for large N, the approximation

$$R_N(t) = n(t)/N \approx R(t) \tag{0.1.8}$$

is valid. To get the same degree of accuracy in our approximation of the function $R(t)$, we need to make considerably more experiments than we would to obtain an approximation for the probability of $R(t_0)$. For this reason, the reliability is often characterized not by the function $R(t)$ only but by certain numerical quantities like the mean and the variance of the lifetime, the availability coefficient and the failure rate.

0.2. LIFETIME AND FAILURE RATE FOR RELIABLE SYSTEMS

One of the important concepts in reliability theory is that of life. The lifetime of a unit means its capacity for extended use under the necessary technological servicing, which may include various types of repairs and maintenances. The lifetime is classified ~~either~~ by time or the number of cycles or the volume of work performed. For certain units, the concepts of life and failure-free operation may coincide, but, in general, these are independent characteristics of reliability. The mean lifetime T of a unit is defined by the mathematical expectation of the random variable τ,

$$\begin{aligned} T = E(\tau) &= \int_0^\infty t f(t)\,dt \\ &= \int_0^\infty t(-R'(t))\,dt \\ &= \int_0^\infty R(t)\,dt. \end{aligned} \tag{0.2.1}$$