STUDY ON THE SCATTERING OF HADRONS BY HEILUM

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A STUDY ON THE SCATTERING OF

HADRONS BY HELIUM

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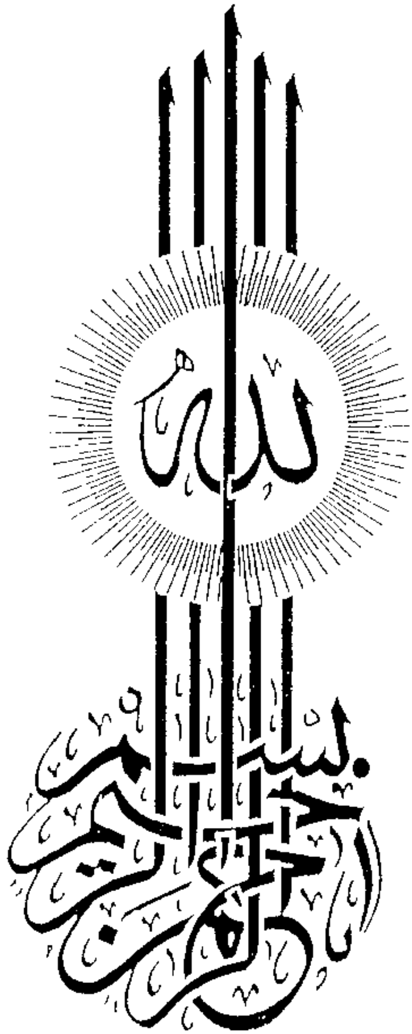

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SUMMARY

This thesis contains a theoretical study on hadron-helium nucleus collision; the diffraction approximation of Glauber (1959) is used. But to obtain the amplitude of elastic scattering , we generalize the method which is suggested before by sitenko (1959) , to study hadron-deuteron collision. This method is generalized to the case of hadron-triton collision by Tartakovsky (1977). Due to the small effect of the spin and the electro-magnetic interaction , we neglect them. Our interest is bound to the case of elastic scattering as we are dealing with the helium nucleus having a high binding energy, and the case of small momentum transfer, which takes place in collision at high energy.

The differential cross section of p-^{4}He elastic scattering at 1 Gev. is calculated. Also we have calculated the total cross section of interaction, at different values of energy.

Our results give a good agreement with experiment. The divergence between the theoretical and experimental results in the region of the momentum transfer of order 0.4 $(Gev/c)^2$ may be taken to be due to the neglect of Coulomb effect which appears clearly in this region.

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INTRODUCTION

An increasing number of experiments have been undertaken in recent years to study the scattering of high-energy particles in nuclei. The electron scattering experiments furnish an accurate determination of the nuclear charge distribution. The use of protons or pions as projectiles in high-energy nuclear scattering experiments [3,2,13,26], has, on the other hand, hardly been more than begun. High-energy data on hardron scattering in nuclei are coveniently analyzed by means of the multiple diffraction theory of Glauber [20,21].

The problem of nuclear diffraction has been discussed by many authors. At the present time, the diffraction method is successfully applied to describe the strong interaction of particles and nuclei with nuclei.

With the appearance of the results[18,19,20] , the theory of diffraction began to develop. the mathematical tool used was similar to that of Fraunhofer's diffraction [24] theory dealing with the interaction of electromagnetic waves of small wave length and with a screen of large modulus , where the approximation of the geometrical optics is valid.

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In[15,16,22] a theoretical analysis of the collisions of particles with deuterons was carried out using the Glauber approach.

In [4,28],Glauber theory was used to study the collision of particles with three-nucleon system. Using Glauber approximation,the high-energy scattering of particles with ^{4}He[7] and with ^{6}Li, ^{7}Li, ^{9}Be, ^{12}C, ^{27}Ac[23] were investigated.

In this thesis, the high-energy collisions of proton with helium are discussed in the framework of the multiple diffraction theory of Glauber and as an extension of the work [28].

In chapter I , some basic concepts of the scattering theory are presented. The amplitude of elastic scattering is obtained. Some of its properties are discussed. The partial wave method is explained.

In Chapter II, the diffraction phenomena of the electromagnetic wave, specially, the Fraunhofer diffraction is investigated.

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In Chapter III , we are concerned with the diffraction approximation in the case of hadron-nucleus collission. The method of derivation of this theory is explained.

In Chapter IV , the theoretical study of hadron-helium elastic scattering is presented. The amplitude of elastic scattering , the differential cross-section of elastic scattering and the totsl cross-section of interaction are calculated.

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CHAPTER 1

Some principles of scattering theory

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The theory of scattering is considered as a very important branch in modern physics. It helps in obtaining many facts of nuclear and atomic physics. In this chapter we present some of its principal concepts which will be used in the following chapters.

1.1 Elastic Scattering

To begin with the simplest type of scattering problem, we shall assume that the incident particles are deflected by a static force field which is localized in range[20]. The field may be represented by a potential $v(\underline{r})$. We shall take the energy of the incident particle to be:

$$E = \frac{\hbar^2 k^2}{2m} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1.1)$$

The symbol $\underline{k}$ will be used to represent the propagation vector of the incident wave. Our problem is to solve the Schroedinger equation.

$$(\nabla^2 + k^2)\, \psi(\underline{r}) = \frac{2m}{\hbar^2}\, v(\underline{r})\psi(\underline{r}) \qquad (1.2)$$

subject to the boundary condition that at large distances from the region occupied by the potential the wave function

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has the asymptotic form

$$\psi(\underline{r}) \sim e^{i\underline{k}\cdot\underline{r}} + f(\theta)\,\frac{e^{ikr}}{r} \qquad \ldots \qquad (1.3)$$

i.e., the sum of the incident plane wave and an outgoing spherical wave with scattering amplitude $f(\theta)$. We choose the incident wave to have unit density so that the incident flux is the incident velocity v. The flux scattered through solid angle $d\Omega$ is just

$$|f(\theta)|^2 \;\frac{1}{r^2}\, v\, r^2 \, d\Omega$$

so that the corresponding differential element , $d\sigma$, of the cross section is given by

$$d\sigma = \frac{\text{Flux through } d\Omega}{\text{Incident flux}} = |f(\theta)|^2 \, d\Omega \qquad \ldots \qquad (1.4)$$

The problem , as we have stated it thus far, falls into two parts. It is necessary to find functions which satisfy a partial differential equation and among these to choose the one satisfying an asymptotic boundary condition. Now for many purposes it is useful to have a more unified formulation of the problem , one which incorporates both the schroedinger

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equation and its boundary condition. Such a statement may be obtained by means of an integral equation. As the first step in formulating an integral equation we define the Green's function, $G(\underline{r},\underline{r}')$, as a solution of the inhomogeneous wave equation

$$(\nabla^2 + k^2)\, G(\underline{r},\underline{r}') = \frac{2m}{\hbar^2}\delta(\underline{r} - \underline{r}') \quad \ldots \qquad (1.5)$$

The similarity of this equation to the Poisson equation permits one to see that the solution has the singularity $1/|\underline{r} - \underline{r}'|$. The general solution may easily be seen to be

$$- \frac{2m}{4\pi\hbar^2}\,\frac{\alpha e^{ik|\underline{r} - \underline{r}'|} + \beta e^{-ik|\underline{r} - \underline{r}'|}}{|\underline{r} - \underline{r}'|}$$

Where $\alpha + \beta = 1$. We shall define $G(\underline{r},\underline{r}')$ to be the amplitude which corresponds to the steady radiation from a coherent source at r' . We , therefore, choose $\alpha = 1$, so that our Green's function is

$$G(\underline{r},\underline{r}') = - \frac{2m}{4\pi\hbar^2}\,\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r} - \underline{r}'|} \quad \ldots \qquad (1.6)$$

Now, it is easy to see that the expression for $\psi(r)$ given by

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$$\psi(\underline{r}) = e^{i\underline{k}.\underline{r}} + \int G(\underline{r}-\underline{r}')\, v(\underline{r}')\psi(\underline{r}')d\underline{r}' \qquad \ldots \quad \ldots \quad \ldots \quad (1.7)$$

satisfies the Schroedinger equation identically. To see if the asymptotic behaviour is correct we expand for large $|\underline{r}| = r$, noting that

$$|\underline{r} - \underline{r}'| \longrightarrow r - \frac{\underline{r}'.\,\underline{r}}{r} ,$$

as the ratio $|\underline{r}'|/r$ approaches zero. The latter ratio is indeed small when r is large since the region of the r' integration extends only over the region where v is different from zero. Now let us define a propagation vector pointing in the direction $\underline{r}$,

$$\underline{k}_r = |\underline{k}| \frac{\underline{r}}{r} = k \frac{\underline{r}}{r} \ldots \qquad (1.8)$$

From the integral equation

$$\psi(\underline{r}) = e^{i\underline{k}.\underline{r}} - \frac{2m}{4\pi\hbar^2} \int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\underline{r} - \underline{r}'|} v(\underline{r}')\psi(\underline{r}')\, d\underline{r}' \qquad \ldots \quad \ldots \quad (1.9)$$

we obtain as $r \longrightarrow \infty$

$$\psi(\underline{r}) \longrightarrow e^{i\underline{k}.\mathbf{r}} - \frac{2m}{4\pi\hbar^2} \frac{e^{ikr}}{r} \int e^{-i\underline{k}_r.\underline{r}'} v(\underline{r}')\psi(\underline{r}')d\underline{r}' . \qquad \ldots \quad (1.10)$$

We see that this has the required asymptotic form. Furthermore,we have obtained as exact expression for the scattering amplitude.

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