CAIRO UNIVERSITY
FACULTY OF ECONOMICS AND POLITICAL SCIENCE

SOME METHODS OF ESTIMATION IN THE CASE OF MULTIVARIATE CAUSES OF FAILURE MODELS

By

MOHAMED ELWY MAHRAN ABDL-HAFEZ

A Thesis Submitted In Accordance With
The Regulation For Candidates For
M. Sc. Degree In Statistics

1981

SOME METHODS OF ESTIMATION IN THE CASE OF MULTIVARIATE CAUSES OF FAILURE MODELS

By

MOHAMED ELWY MAHRAN ABD-EL-HAFEZ

Supervised By

Dr. Ahmed Mohamed Omar
Professor Of Statistics,
Faculty of Economics And
Political Science, Depa-
rtment Of Statistics,
Cairo University.

Dr. Samir Kamel Ashour
Lecturer of Mathematical
Statistics at The Instit-
ute of Statistical Studies
and Research, Cairo
University.

For

The Degree Of Master Of Science
In Statistics

بسم الله الرحمن الرحيم

APPROVAL SHEET

SOME METHODS OF ESTIMATION IN THE CASE OF MULTIVARIATE CAUSES OF FAILURE MODELS

BY

MOHAMED ELWY MAHRAN ABDL-HAFEZ

This thesis for the Master degree of Science in Statistics, has been approved by:

Nadia Makary Girgis

Ahmed Mawaziny

Ahmed Omar

TO MY GOD

ACKNOWLEDGEMENT

I am deeply indebted to Professor. Dr. Ahmed Mohamed Omar, Professor of Statistics in Faculty of Economics and Political Science, Department of Statistics, Cairo University, for his encouragements and helpful facilities he gave during the course of study.

I would like to convey my profound gratitude to Dr. S.K. Ashour, Lecturer of Mathematical Statistics at the Institute of Statistical Studies and Research, Cairo University, for his creative suggestions, valuable help and guidance throughout the supervision of this thesis.

My deep thanks go to all those who have helped in a way or another in the completion of this thesis.

SUMMARY

This thesis is concerned with the problem of estimating the unknown parameters of compound and mixed models if the underlying distributions are exponential and Weibull. We shall review some statistical failure models, such as exponential and Weibull distributions for single cause of failure models and mixed and compound models for multicause of failure models. Also we shall review the methods of estimation for the compound and mixed models, and present the known properties of the estimators and some numerical investigations .

For mixed Weibull with equal shape parameter, since the Bayesian method had not been discussed previously we shall present a full Bayesian analysis for the estimation problem of the unknown parameters using complete and censored data.

CONTENTS

CHAPTER (I)

INTRODUCTION

In many practical situations in the field of life testing, we have one potential cause of failure, while in others there are more than one potential cause of failure for all items on a test. In the case of multi-causes of failure, the problem of estimating the unknown parameters for the compound and mixed models if the underlying distributions are exponential and Weibull has been considered by many authors. For compound exponential distribution, Bayesian and non-Bayesian estimates have been obtained by Boardman and Kendell (1970), Herman and Patell (1971), Boardman (1973), Bloc and Basu (1974) and Bancorft and Dunsmore (1976), while for compound Weibull distribution, the same problem has been considered by David and Moeschberger (1971), Herman and Patell (1971), Moeschberger (1974) and Ashour, Shoukry and Mohamed (1979). Similarly, for mixed exponential distribution, Bayesian and non-Bayesian estimates have been obtained by Mendenhall and Hader (1958), Rider (1960), Swamy and Doss (1960), Tallis and Light (1967) and Ashour and Jones (1976) while for mixed Weibull distribution, Kao (1959), Falls (1970) and Ashour and Jones (1976) obtained non-Bayesian estimates for the unknown parameters.

In the present thesis, a comperhensive review for all methods of estimation for mixed and compound models will be presented. A compartive study between all these estimates has been considered. Some random numbers for exponential and Weibull distributions have been generated and tested and can be used to check any, iterative, procedure.

For mixed Weibull model, since Bayesian method of estimation had not been discussed previously, we shall present a full Bayesian analysis to estimate the unknown parameters for mixed Weibull with equal shape parameters in the case of complete and censored data. Using dependent and independent priors for the the unknown parameters, the joint posterior distribution and its trivariate, bivariate, and univariate marginal densities will be derived in the case of informative and non-informative priors. These distributions may be used to obtain Bayes' estimates.

Using univariate marginal densities, squared error loss, iterative procedure, and computer facilities, a numerical example will be carried out in the case of informative and non-informative prior.

Clearly, the results in the case of mixed exponential and simple Weibull and exponential can be obtained directly as special cases from our results. Also our results can be extended to the case of multicauses of failure (more than two causes).

This thesis consists of six chapters, chapter II consists of some definitions and notation which will be used in the present thesis, chapter III consists of some statistical failure models, chapter IV deals with the estimation problem for compound model, chapter V deals with the same problem for mixed model, while chapter VI concerns with the Bayesian estimation for mixed Weibull model.

Definition (2.1)

A sample is truncated if it is drawn from an incomplete population.

(2.3) Censored Sample

Suppose that due to restrictions on time available for testing, the experimenter frequently desires to conclude the life-test after a predetermined conditions. Sampling of this type is known as censored sampling. Bassett (1964) gave the following definition

Definition (2.2)

A sample is said to be censored if while it is drawn from complete population, some of the individual values of its members are unknown, such sample members are themselves called censored. Censored sample can be classified into two types: single and doubly censored sample.

(2.3.1) Single Censored Sample

In single censored sample we consider the case in which we sample from complete population but either the last few observations (in which case censoring is on the right) or the first few observations (in which case censoring is on the left) are unknown. A type I censored above sample is one in which the test is terminated at specified time T, before all the items, n say, have failed or sooner if all items have censoring occurs when the number of survivors drops to predetermined levels.

Let n designates the total sample size and k the number of sample specimens which fail and which therefore results in completely determined life spans. Suppose that censoring occurs progressively in h stages at times T_i such that $T_i > T_{i-1}$, i=1,2,...h, and that at the ith stage of censoring, r_i sample specimens selected randomly from the survivors at time T_i are removed from further observation. It follows that

$$n = k + \sum_{i=1}^{h} r_i \qquad (2.1)$$

In type I censoring, the T_i are fixed and the number of survivors at these times are random variables. In type II censoring the T_i coincide with times of failure and are random variables, whereas the number of survivors at these times are fixed. For both types the r_i's are fixed.

(2.3.2) Doubly Censored Sample

Consider the case in which we sample from complete population but censoring is from two sides, i.e, the first few observations and the last few observations are unknown. Thus the incomplete lifetimes from two sides designated as doubly censored observations.

There are also two types of doubly censored samples;

type I and type II; as in single censored sample. In type I censoring, the times T_1 and T_2 where $T_1 < T_2$ at which censoring occurs are fixed, while k_1 small and k_2 large sample members unobserved are random variables. In type II consoring the smallest k_1 and the largest k_2 censored observations are predetermined while $x_{(k_1+1)}, x_{(n-k_2)}$ are random variables and there are $(n-k_1 - k_2)$ uncensored observations. The case of single censoring is a special case of the doubly censoring and can be obtained by putting $k_1 = 0$ or $k_2 = 0$.

(2.4) Hazard Rate

Let F(x) be the distribution function of the time to failure random variable, x, and let f(x) be its probability density function. Then the hazard rate, h(x), can be defined as:

$$h(x) = \frac{f(x)}{1 - F(x)} \qquad (2.2)$$

where 1-F(x) is called the raliability at time x. Barlow and Proschan (1965) gave a probabilistic interpretation for the hazard rate, namely, h(x) dx which represents the probability of failure in a finite interval of time $(x, x + \Delta x)$ given that the age of the components is x. The hazard rate by this definition would be:

$$\frac{F(x + \Delta x) - F(x)}{1 - F(x)} \qquad (2.3)$$