

A Thesis on
NUCLEAR REACTIONS AT HIGH ENERGIES

Submitted by

Magda Talib Aly Hilal Al-Hinai

B.Sc. 1991

Supervised by

PROF. DR. M. G. EL MOHANDIS

PROF. DR. A. Y. ABUL-MAGD

Department of Mathematics,
Women's University College for Arts,
Science and Education,
Ain Shams University.

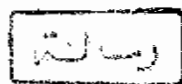
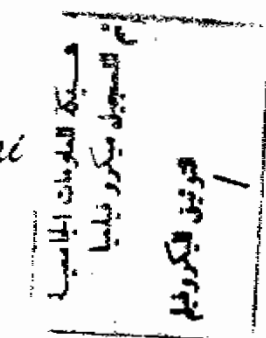
Department of Mathematics,
Faculty of Science,
Zagazig University.

To

Women's University College for Arts, Science and Education
Ain Shams University

In Partial Fulfilment of the Requirements for the Award of the
M.Sc. Degree in Applied Mathematics

1995



٥٨٠٦٤



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ
قَالَ سَنَشُدُّ عَضُدَكَ بِأَخِيكَ وَنَجْعَلُ
لَكُمَا سُلْطَانًا فَلَا يَصِلُونَ إِلَيْكُمَا
بِأَيِّتِنَا أَنْتُمَا وَمَنْ اتَّبَعَكُمَا الْغَالِبُونَ
صَدَقَ اللَّهُ الْعَظِيمُ



*with gratitude
I dedicate this work
to the memory of
my dear father*



Acknowledgement



*Thank God for graciously granting
me the privilege of working
under the supervision of
Prof. Dr. M. G. El Mohandis
and Prof. Dr. A. Y. Abul-Magd,
who were very patient, gentle and
understanding in their guidance.*

*Thank God for mercifully supporting
me with the encouragement of all the
staff of the Mathematics Department,
Women's University College for
Arts, Science and Education.*

*Thank God for generously offering
me all benefits, a wise mother and
a magnanimous brother always
busy caring for me presenting
all facilities and help.*

NOTE

In addition to the research work covered in this thesis, the candidate has passed written examinations in the following postgraduate courses:

- Quantum Mechanics.
- Group Theory.
- Theory of Atomic Collisions.
- German Language.

with grade “Excellent” in each.

TABLE OF CONTENTS

ACKNOWLEDGEMENT

ABSTRACT

INTRODUCTION 1

CHAPTER I : A REVIEW OF GLAUBER'S THEORY 6

THE HIGH ENERGY APPROXIMATION. 8

NUCLEUS-NUCLEUS COLLISIONS. 22

CHAPTER II FORMULATION AND PREDICTION OF THE REACTION CROSS SECTION IN NUCLEAR COLLISIONS. 32

FORMALISM. 32

THE REACTION CROSS SECTION FOR PROTON NUCLEUS COLLISIONS. 32

 The Gaussian distribution of the target nuclear density. 35

 The uniform distribution of the target nuclear density. 36

 Wood- Saxon's nuclear density distribution. 37

THE REACTION CROSS SECTION FOR NUCLEUS-NUCLEUS COLLISIONS. 39

 The reaction cross section for collisions between two nuclei having Gaussian
 density distribution 40

 The reaction cross section between a projectile having a Gaussian density
 distribution and a target with a uniform density distribution 41

CALCULATIONS AND DATA ANALYSIS. 44

 IN PROTON-NUCLEUS COLLISIONS. 44

 IN NUCLEUS-NUCLEUS COLLISIONS. 57

CHAPTER III : MODIFICATION OF THE GLAUBER MODEL

DESCRIPTION OF THE REACTION CROSS SECTION **67**

COULOMB MODIFIED GLAUBER MODEL.	67
THE OPTICAL LIMIT OF GLAUBER'S THEORY.	68
THE MODIFIED GLAUBER MODEL FOR THE DESCRIPTION OF COLLISIONS BETWEEN HEAVY NUCLEI.	74
THE COULOMB MODIFIED GLAUBER MODEL DESCRIPTION OF THE REACTION CROSS SECTION.	78
CALCULATIONS AND DATA ANALYSIS.	80
THE COULOMB MODIFIED REACTION CROSS SECTION	80

CHAPTER IV THE DISASSEMBLY OF HEAVY NUCLEI BY HIGH

ENERGY PROTONS **88**

INTRODUCTION	88
FORMALISM	97
CALCULATIONS AND DATA ANALYSIS	100
THE FIRST STEP OF THE REACTION	100
THE SECOND STEP OF THE REACTION	103

APPENDICES **109**

THE REACTION CROSS SECTION FOR GAUSSIAN-GAUSSIAN NUCLEAR DENSITIES	109
THE OVERLAP INTEGRAL OF GAUSSIAN-UNIFORM NUCLEAR DENSITIES.	113

CONCLUSIONS **116**

REFERENCES **117**

ARABIC SUMMARY **120**

LIST OF FIGURES

FIG 1.1	AN ILLUSTRATIVE FIGURE SHOWING THE TWO COMPONENTS ($\bar{b}$, z) OF THE POSITION VECTOR $\bar{r}$	9
FIG. 1.2	A SCHEMATIC REPRESENTATION OF HIGH ENERGY NUCLEUS-NUCLEUS COLLISIONS.	23
FIG. 2.1	PROTON-NUCLEON CROSS SECTIONS.	46
FIG. 2.2	PROTON-NUCLEUS REACTION CROSS SECTIONS CALCULATIONS FOR ${}^9\text{Be}$.	50
FIG. 2.3	PROTON-NUCLEUS REACTION CROSS SECTIONS CALCULATIONS FOR ${}^{12}\text{C}$.	51
FIG. 2.4	PROTON-NUCLEUS REACTION CROSS SECTIONS CALCULATIONS FOR ${}^{16}\text{O}$.	52
FIG 2.5	PROTON-NUCLEUS REACTION CROSS SECTIONS CALCULATIONS FOR ${}^{27}\text{Al}$.	53
FIG. 2.6	PROTON-NUCLEUS REACTION CROSS SECTIONS CALCULATIONS FOR ${}^{65}\text{Cu}$.	54
FIG 2.7	PROTON-NUCLEUS REACTION CROSS SECTIONS CALCULATIONS FOR ${}^{208}\text{Pb}$.	55
FIG. 2.8	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR ${}^{12}\text{C}+{}^{12}\text{C}$.	59
FIG 2.9	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR ${}^{20}\text{Ne}+{}^{12}\text{C}$.	60
FIG 2.10	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR ${}^{12}\text{C}+{}^{27}\text{Al}$.	61
FIG. 2.11	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR ${}^{12}\text{C}+{}^{57}\text{Fe}$.	62
FIG. 2.12	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR ${}^{12}\text{C}+{}^{65}\text{Cu}$.	63
FIG. 2.13	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR ${}^{12}\text{C}+{}^{66}\text{Zn}$.	64

FIG. 2.14	NUCLEUS-NUCLEUS REACTION CROSS SECTION CALCULATIONS FOR $^{12}\text{C}+^{89}\text{Y}$.	65
FIG. 3.1	ELASTIC DIFFERENTIAL CROSS SECTION FOR $^{13}\text{C}+^{208}\text{Pb}$.	73
FIG. 3.2	DIAGRAMMATIC REPRESENTATION OF COLLISIONS IN THE PRESENCE AND THE ABSENCE OF THE COULOMB EFFECTS	74
FIG. 3.3	THE ANGULAR DISTRIBUTION FOR $^{40}\text{Ar}+^{208}\text{Pb}$ AND $^{13}\text{C}+^{208}\text{Pb}$.	77
FIG. 3.4	THE COULOMB MODIFIED REACTION CROSS SECTION PREDICTIONS FOR $p+^{12}\text{C}$.	83
FIG. 3.5	THE COULOMB MODIFIED REACTION CROSS SECTION PREDICTIONS FOR $p+^{40}\text{Ca}$.	84
FIG. 3.6	THE COULOMB MODIFIED REACTION CROSS SECTION PREDICTIONS FOR $p+^{208}\text{Pb}$.	85
FIG. 3.7	THE COULOMB MODIFIED REACTION CROSS SECTION PREDICTIONS FOR $^{12}\text{C}+^{12}\text{C}$.	86
FIG. 3.8	THE COULOMB MODIFIED REACTION CROSS SECTION PREDICTIONS FOR $^{16}\text{O}+^{208}\text{Pb}$, $^{20}\text{Ne}+^{236}\text{U}$, $^{20}\text{Ne}+^{208}\text{Pb}$ AND $^{40}\text{Ar}+^{109}\text{Ag}$	87
FIG. 4.1	AN ILLUSTRATION OF THE KNOCK-ON CASCADE INITIATED BY A HIGH ENERGY PROTON.	88
FIG. 4.2	A DESCRIPTION OF THE DISINTEGRATION OF A NUCLEUS SHOT BY A HIGH ENERGY PROTON.	89
FIG. 4.3	CALCULATIONS OF THE CROSS SECTION σ , FOR SEVERAL PROTON NUCLEUS COLLISIONS.	101
FIG. 4.4	THE DISTRIBUTION OF THE EXCITATION ENERGY FOR $p+^{159}\text{Tb}$ REACTION	102
FIG. 4.5	THE EXCITATION ENERGY LOST PER UNIT MASS EVAPORATION FOR Cu, Ag AND Au.	105
FIG. 4.6	THE MASS YIELD CURVES FOR $p+\text{Cu}$ AND $p+\text{Ag}$ AT 3 GeV.	106
FIG. 4.7	THE MASS YIELD CURVES FOR $p+\text{Au}$ AT 1, 3 AND 6 GeV.	107
FIG. 4.8	THE MASS YIELD CURVES FOR $p+\text{Ta}$ AT 5.7 GeV AND $p+\text{Ag}$ AT 300 GeV.	108

LIST OF TABLES

TABLE 2.1 THE NUCLEON-NUCLEON CROSS SECTION IN THE ENERGY RANGE 100-2200 MeV. _____	47
TABLE 2.2 THE VALUES OF THE DIFFUSENESS AND THE CENTRAL NUCLEAR DENSITY FOR GAUSSIAN DISTRIBUTIONS. _____	48
TABLE 2.3 A COMPILATION OF THE WOOD-SAXON DENSITY PARAMETERS. _____	49
TABLE 2.4 A COMPARISON OF THE PROTON-NUCLEUS REACTION CROSS SECTION PREDICTIONS WITH EXPERIMENT. _____	56
TABLE 2.5 A COMPARISON OF THE NUCLEUS-NUCLEUS REACTION CROSS SECTION PREDICTIONS WITH EXPERIMENT. _____	66

ABSTRACT

Abstract

High energy nuclear reactions are treated by Glauber's multiple scattering theory. Within the framework of this theory, a major part of this thesis is devoted for the study of the total reaction cross section, σ_R , and its modifications. As a primary goal, the reaction cross section for proton-nucleus collisions is calculated. The following target nuclear density distributions are adopted during calculations: the Gaussian distribution, the uniform distribution, and finally, the two-parameter Fermi distribution (Wood-Saxon). For the Gaussian and the uniform nuclear shapes, the reaction cross section is analytically derived. Applying the Wood-Saxon distribution, the problem is intractable analytically and thus σ_R is numerically evaluated. The following targets are considered ^9Be , ^{12}C , ^{16}O , ^{27}Al , ^{65}Cu and ^{208}Pb , where the projectile energy varied from 100 to 2200 MeV. Also, a compiled comparison of the reaction cross section experimental results with the present calculations is presented for a variety of targets at different proton energies.

In nucleus-nucleus collisions, firstly, the reaction cross section is analytically calculated for interactions between nuclei with Gaussian nuclear density distributions. Another case is considered in which the projectile density distribution is Gaussian while the target density is assumed to be uniform. The following reactions are considered: $^{12}\text{C} + ^{12}\text{C}$ at energies 50 - 1000 MeV/nucleon, $^{20}\text{Ne} + ^{12}\text{C}$, $^{12}\text{C} + ^{27}\text{Al}$, $^{12}\text{C} + ^{57}\text{Fe}$, $^{12}\text{C} + ^{66}\text{Zn}$ at energies 50 - 350 MeV/nucleon, and $^{12}\text{C} + ^{56}\text{Cu}$ at energies 50 - 1500 MeV/nucleon. in addition to a tabulated comparison of the reaction cross section predictions with the corresponding experimental results for several nucleus-nucleus reactions at

different energies. The present calculations of the reaction cross section are shown to be in close agreement with experiment.

In the second stage of this work, the Glauber description of the reaction cross section is modified to describe low energy reactions. This modification diminishes some deviations that occur at low energies.

In the low energy region, where the Coulomb effects are significant, an expression that relates the reaction cross section in the presence of the Coulomb effects to the reaction cross section in the absence of these effects is established.

Finally, the thesis is ended by the discussion of high energy proton induced spallation reactions which are viewed as two step processes. In the first step, and for a Gaussian target nucleus, an analytical expression is derived for the cross section σ_n for the incident proton to collide with n of the target constituents. The following targets are considered: ^{12}C , ^{27}Al , ^{64}Cu and ^{208}Pb to estimate the dependence of σ_n on n . In the same step of the reaction, the cross section $\frac{d\sigma}{dE^*}$ to deposit an energy E^* , after n proton-nucleon collisions, is calculated for the $p+^{159}\text{Tb}$ collision at $E_p=600\text{MeV}$. In the second step of the reaction -the decay of the struck target- the mass yield distribution is predicted for the following reactions: $p + \text{Cu}$, $p + \text{Ag}$ at $E_p = 3 \text{ GeV}$, $p + \text{Au}$ at $E_p = 1 \text{ GeV}$, 3 GeV and 6 GeV , $p + \text{Ta}$ at $E_p = 5.7 \text{ GeV}$ and $p + \text{Ag}$ at $E_p = 300 \text{ GeV}$. The predictions provide a reasonable representation of the available experimental data.

INTRODUCTION