NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS

THESIS
SUBMITTED FOR THE DEGREE
OF
DOCTOR OF PHILOSOPHY IN SCIENCE
(Pure Mathematics)

PRESENTED BY

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SUBMITTED TO

Department of Mathematics
Faculty of Science,
Ain Shams University
Cairo, Egypt.
1996

بسم الله الرحمن الرحيم

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SUMMARY

It is our aim to demonstrate the great importance of Partial differential equations as the mathematical language for describing the physical phenomena, sediment transport in rivers, as well as the importance of numerical methods in solving these equations.

Why should one study partial differential equations?. The traditional answers are that partial differential equations have important applications outside of mathematics, and that they are the proper source of many problems and concepts in Functional Analysis.

Why should one study numerical methods for solving partial differential equations?. The traditional answers are that sometimes numerical methods are a short way to get an approximate answer and that sometimes it is difficult or impossible to obtain the analytical solution. A third possible answer is the progress in: the computer manufacturing, the computer systems, the Artificial Intelligence Packages and the Parallel Computations which simplified and accelerated the calculation processes and sometimes numerical methods are the only possible method!

Among other numerical methods for solving partial differential equations the finite difference methods and the finite element methods are of great importance, the philosophy of both of them is the replacement of equations and systems of partial differential equations by another system of algebraic or ordinary differential equations which can be solved efficiently even with the available packages. Finite difference is the oldest but now finite element is well established. The advantages of the finite element method lies in a higher flexibility in particular for: (i) the representation of arbitrary complicated boundaries, (ii) systematic rules for the developments of stable numerical schemes approximating mathematically well posed problems, with various types of boundary conditions, but this flexibility is paid by: an increased programming complexity, additional storage requirement compared

to the finite difference method. Also, it is possible to use both of the finite difference and the finite element methods in the same problem, the method of discretization in time, Rektorys [45].

The aim of this thesis is mathematical modeling of sedimentation and numerical solution of the resulting system of partial differential equations. The thesis consists of four chapters:

CHAPTER I :

In this chapter we have considered the problem of sediment transport from the physical point of view and we have illustrated the coupling between the sediment movements and the motion of the carrying fluid. Also, we have considered the forces acting on the particles and the resulting movements for non-cohesive sediments, and we also, considered the cohesive case.

CHAPTER II:

In this chapter, we considered a general mathematical model, we have used the Navier-Stokes equations to describe the three dimensional fluid motion and a system of continuity equations to describe the sediment motions. We mentioned the shallow water assumptions, considered only one continuity equation for sediments and introduced a general form for the boundary conditions i.e. the three dimensional model. Also, we mentioned the difficulties in solving this model. We also introduced a complete description for the two dimensional model and we have showed how this model can be obtained from the corresponding three dimensional one through an averaging process.

CHAPTER III:

Our aim is the numerical solution of the model introduced in Chapter II. We considered in this chapter the classical numerical methods (Ritz & Galerkin's) also we considered the variational formulation and the weak forms of partial differential equations and we have studied the classification of boundary conditions as