AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING

# STUDY OF PLATE BENDING ELEMENT SUITABLE FOR THICK AND THIN PLATES

BY

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## STATEMENT

The dissertation is submitted to Faculty of Engineering, Ain Shams University for degree of MASTER OF SCIENCE in Civil Engineering ( Structural Division ).

The work included in this thesis was carried out by the author in the Department of Structural Engineering, Faculty of Engineering, Ain Shams University, from 1992 to August 1995.

No part of this thesis has been submitted for a degree or a qualification to any other university or institution.

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## Structural Department

Abstract for master of science thesis by Eng. **Yasser Galal Eldin Mohamed Fahmy** .

*Title* : Study of plate bending element suitable for thick and thin plates.


## ABSTRACT

The finite element method is a powerful numerical technique for the analysis of plates. However, direct applications of the *REISSNER-MINDLIN* plate theory for the derivation of plate elements suitable for analyzing thick and thin plates lead to very disappointing results.

The purpose of this research is to derive a plate bending element valid for thick and thin plate situations. The condition used for the derivation of this plate element is the vanishing of the shear strain for the thin plate limit. This is achieved by defining the approximating shear strain polynomial as a linear function for both nodal rotations and deflections.

A simple explicit form of the substitute shear strain matrix is, thus, obtained . The general methodology is applied to the re-formulation of some well-known 4 and 9 node quadrilateral plate elements .

Computer programs have been developed for 4, 8 and 9 noded quadrilateral elements based on the formulation mentioned above . Some numerical examples demonstrate the use of the developed technique to improve the behavior of the elements. Comparison between the results from this technique and other methods is given to show the advantage of the new formulation. The performance of the 9-node Lagrangian element with the new technique is near to the optimal.


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# TABLE OF CONTENTS

*Chapter* *page*

*Chapter* *page*

* * *

# LIST OF FIGURES

* * *

# LIST OF TABLES



* * *

# LIST OF NOTATIONS

| Symbol | Definition |
|---|---|
| [ A ] | Nodal displacement-constants matrix |
| [ B ] | Strain-displacement matrix |
| [ $B_b$ ] | Curvature-displacement matrix |
| [ $B_s$ ] | Standard shear strain matrix |
| $\left[\hat{B}_s\right]$ | Substitute shear strain matrix |
| [ C ] | Cartesian-natural shear strain matrix at sampling points |
| [ CN ] | Nodal coordinate matrix |
| [ D ] | Diagonal matrix |
| [ $D_b$ ] | Moment-curvature matrix |
| [ $D_s$ ] | Shear force-shear strain matrix |
| [ E ] | Stress-strain matrix |
| E | Elastic modulus |
| G | Shear modulus |
| [ J ] | Jacobian matrix |
| [ K ] | Element stiffness matrix |
| { M } | Generalized bending moment vector |
| $M_x$ , My , Mxy | Components of bending moment |
| [ N ] | Shape function matrix |
| { P } | Nodal load vector |
| [ PF ] | Partial derivatives w.r.t local coordinates matrix |
| { Q } | Transverse shear force vector |
| $Q_x$ , Qy | Components of transverse shear force |
| [ T ] | Transformation matrix |
| { U } | Generic displacement vector |
| [ U ] | Upper triangle matrix |
| b | Subscript denoting bending |
| { b } | Body force vector for element |
| d | Linear differential operator |
| e | Superscript denoting element |
| f | Function |
| i | Index |
| j | Index |

| Symbol | Definition |
|---|---|
| k | Index |
| n | Number |
| $\{ q \}$ | Nodal displacement vector |
| s | Subscript denoting shear |
| t | Thickness of plate |
| u | Translation in x direction |
| v | Translation in y direction |
| w | Translation in z direction |
| x | Cartesian coordinate |
| y | Cartesian coordinate |
| z | Cartesian coordinate |
| $\alpha$ | Warping coefficient |
| $\{ \alpha \}$ | Constants vector |
| $\{ \gamma \}$ | Shear strain vector |
| $\gamma_x , \gamma_y$ | Components of shear strain |
| $\{ \varepsilon \}$ | Normal strain vector |
| $\varepsilon_x , \varepsilon_y$ | Components of normal strain |
| $\eta$ | Natural coordinate |
| $\{ \theta \}$ | Rotation vector |
| $\theta_x , \theta_y$ | Components of rotation |
| $\nu$ | Poisson's ratio |
| $\xi$ | Natural coordinate |
| $\{ \sigma \}$ | Normal stress vector |
| $\sigma_x , \sigma_y$ | Components of normal stress |
| $\{ \tau \}$ | Shear stress vector |
| $\tau_{xz} , \tau_{yz}$ | Components of shear stress |
| $\tau_{xy}$ | Shear stress in plane |
| $[ \phi ]$ | Generic displacement-constant matrix |
| $\{ \chi \}$ | Curvature vector |
| $\chi_x , \chi_y , \chi_{xy}$ | Components of curvature |

*Chapter (1)*

# INTRODUCTION