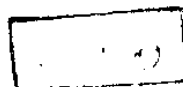


**ON INTEGRODIFFERENTIAL EQUATIONS
AND MATHEMATICAL MODELS IN
POPULATION DYNAMICS**

THESIS
Submitted For The Degree
OF
DOCTOR OF PHILOSOPHY (Ph.D.)
IN
Pure Mathematics

BY



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Title of Thesis

**ON INTEGRODIFFERENTIAL EQUATIONS AND
MATHEMATICAL MODELS IN POPULATION
DYNAMICS**

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ABSTRACT

SABAH HAFEZ ABDALLAH. On integrodifferential equations and mathematical models in population dynamics. Unpublished Doctor of philosophy dissertations of pure mathematics , University College for, Women , Ain Shams University.

The main purpose of this dissertation is to improve results for the oscillations of neutral differential equations of odd order. Also some new comparison results are obtained.

A model of stage - structured population growth, where the age to maturity is state dependent, is investigated. The stability of positive equilibrium criteria is studied.

New necessary and sufficient conditions for all solutions of an integrodifferential equations to have at least one zero crossing is obtained. An application to a Logistic integrodifferential equation is discussed.

We introduce , new results for global asymptotic stability in a competition system of four competitive species modelled by autonomous delay integrodifferential equations are obtained . We also introduce sufficient conditions for persistence and stability of an autonomous integrodifferential system.

Key Words : Neutral differential equations, population dynamics, Logistic equations , oscillations, zero crossing , stage-structure, time delay, equilibrium, stability, asymptotic stability, persistence, Lyapunov functional.

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SUMMARY

