DIFFERENTIAL - DIFFERENCE EQUATIONS AND RELATED TOPICS

Thesis

Submitted for the Degree of Doctor of Philosophy (Ph. D) Pure Mathematics

by

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CONTENTS

SUMMARY

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This thesis deals with the differential-difference equations and related topics. It aims at numerical treatment of neutral differential-difference equations. We are concerned with numerical multistep methods for solving neutral differential-difference equations. We use the concept of continued fractions for solving a class of differential-difference equations, and a class of difference equations. The thesis consists of six chapters.

In chapter (1), we have studied the importance of difference equations and a number of problems from which difference equations arise. We study the solution of linear difference equations with constant coefficients of n^{th} order. We also study systems of first order linear difference equations with variable coefficients. We could transform a higher order difference equation into a system of first order difference equations.

In chapter (2), we have studied the general form of the differential-difference equations and basic definitions. We have studied the fundamental theorems for the existence and uniqueness of solutions of first order equation of neutral type,

$$a_o y'(t) + a_1 y'(t-\omega) + b_o y(t) + b_1 y(t-\omega) = f(t), \; a_o \neq 0, \; a_1 \neq 0,$$

with the initial function

$$y(t) = g(t), \qquad 0 \leq t \leq \omega,$$

and the condition of continuity of the derivative of the solution at $t=\omega$. We could derive the priori estimate of the norm of the solution of a system of neutral differential-difference equations. We have studied the characteristic matrix function and series expansions of

the system. We discuss the transform of a higher order neutral differential-difference equation to a system of first order equations.

The presentation in chapter (3) is centered on the numerical treatment for solving the neutral differential-difference equations:

$$y'(t) = F(t, y(\cdot), y'(\cdot)), \qquad t \in [a, b]$$

$$y(t) = g(t), \qquad t \in [\alpha, a]$$

We have studied the existence of solution of the neutral differential-difference equations and discussed a necessary and sufficient condition for convergence of a general one-step method.

In this chapter, we present methods for the numerical solution of these type of equations in the special form

$$y'(t) = F(t, y(t), y(t-\omega(t)), y'(t-\omega(t))), \ t \in [a,b]$$

$$y(t) = g(t), \qquad t \in [a-\omega(a), a]$$

and we studied the convergence of the method. Also, it has been shown that the linear multistep methods used for ordinary differential equations can also be used to generate the solution of neutral differential-difference equations.

We apply the modified predictor corrector to the NDDEs for the work which has been published in [46], the proceeding of the fifth Conference on Operations Research and its Military Applications, PP. 448-462, Cairo, Egypt, 1993. A numerical examples are given.

In chapter (4), we have studied definitions and basic properties of the continued fractions. We used the concept of continued fractions and their applications in different topics. We have treated some linear difference equations and differential-difference equations. We have studied the connection between continued fractions and sequence that satisfies a system of three-term recurrence relations. We have studied the relation between the infinite series and the continued fractions. At the end of the chapter we have studied the convergence theory of continued fractions.

In chapter (5), we use the relation of the continued fractions to the theory of three-term recurrence relations. The continued fractions may be used in an algorithm for solving a class of second order difference equations with variable coefficients

$y_n + a_n y_{n+1} + b_n y_{n+2} = 0$, the integer $n : n \geq 0$ $b_n \neq 0$

where a_n and b_n are functions of n.

Algorithm-based on continued fraction is mathematically equivalent to the Miller algorithm but for which overflow can be avoided. Application for this algorithm to the model of traffic in channels is considered and comparison between the two algorithms is discussed and numerical examples are given.

This work has already been published in [45] the proceeding of the 20th International Conference of Computer Science and Statistics PP. 27-40. Cairo, 1995.

In chapter (6), using the relation between the continued fractions and power series expansions, we presented a new approach for solving a class of infinite number of differential-difference equations

$$y_0'(t) = -\lambda_0 y_0(t) + \mu_1 y_1(t) ,$$
$$y_r'(t) = \lambda_{r-1} y_{r-1}(t) - (\lambda_r + \mu_r) y_r(t) + \mu_{r+1} y_{r+1}(t), \quad r = 1,2,\ldots$$

where λ_r and μ_r are positive functions of r and independent of t. Also, we studied how quotient-difference algorithm can play an important role in solving the differential-difference equations. We obtained a continued fractions representing the Laplace transform of the solution and computational procedures for computing the solution based on the quotient-difference algorithm. Application for this method to the model of traffic in channels is considered and some tables for solving this model are given.

This work has already been published in [44] the proceeding of the 19th International Conference of Computer Science and Statistics, PP. 9-23, Cairo, 1994.

Another application of continued fractions to the gas station problem is solved.

The Micro-VAX Computer (VT.3100) used for computing all numerical results is given in this thesis.

CHAPTER (1)

LINEAR DIFFERENCE EQUATIONS

CHAPTER (I)

LINEAR DIFFERENCE EQUATIONS

1.1. Introduction

Difference Equations play an important role in pure and applied mathematics such as combinatorial analysis, the theory of probability, and mathematical economics.

Various problems arising in mathematics, physics, engineering and other sciences can be formulated by the use of difference equations.

In recent years there has been an increasing lot, of information in the treatment of finite difference equtions. The advent of high speed computers has led to a need for fundamental knowledge of the difference equations. There are numerous applications of this subject to fields ranging from engineering, physics, and chemistry to actuarial science, economics, psycholagy, probability and statistics. The mathematical theory is of interest in itself especialy in view of the remarkable analagy of the theory to that of differential and integral calculus and differential equations.

All one step methods or multistep methods for solving I.V.P for ordinary differential equations or partial differential equations will take the forms of difference equations.

For solving a system of k simultaneous first order differential equations in the form:

$\frac{dY}{dx} = F(x,Y), Y(x_0)=Y_0$ and $x \in [x_o, x_e]$, we can apply an explicit or a pair of q steps explicit and implicit formulas as:

$$\sum_{i=0}^{q} \alpha_i Y_{n+i} = h \sum_{i=0}^{q} \beta_i F_{n+i} \quad \text{and}$$

$$n=0,1,2,\ldots$$

$$\sum_{i=0}^{q} \alpha_i^* Y_{n+i} = h \sum_{i=0}^{q+1} \beta_i^* F_{n+i},$$

where $F_m = F(x_0 + mh, Y_m)$

which q and q+1 th order linear difference equations.

Difference equations can be used in evaluating particular determinants which appear in the solution of partial differential equations.

Many integral problems which are of great importance in practice, can not be evaluated formally in closed forms. These problems are transfomed to difference equations with variable coefficients in general.

1.2. Linear Difference Equations of First Order

The general first order linear difference equation in n:

$$y_{n+1} - P_n y_n = q_n, \tag{1.1}$$

has the general solution [41], [30]

$$y_n = \left(\prod_{1}^{n-1} p_n\right)\left\{C + \sum_{1}^{n-1}\left(q_n \Big/ \prod_{1}^{n} p_n\right)\right\}, \tag{1.2}$$

where p_n and q_n are finite and defined $\forall\, n \in I^+$, $p_n \neq 0\ \forall\, n \in I^+$, I^+ is the set of all positive integral numbers, C is an arbitrary constant, and it can be replaced by an arbitrary unit periodic function ω_n.

1.3. Nth Order Linear Difference Equations with Constant Coefficients

In this section we shall study the general solution of the nth order linear difference equations of the form: