Efficiency of the Spectral Wavelets Approach in Solving Systems of Differential Equations

By

Mahmoud Mohamed Mokhtar

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Supervisors

Prof. Dr. N. H. Sweilam
Department of Mathematics
Faculty of Science
Cairo University

Dr. I. K. Youssef
Department of Mathematics
Faculty of Science
Ain Shams University

Dr. A. M. Nagy
Department of Mathematics
Faculty of Science
Benha University

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APPROVAL SHEET

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Name of the candidate:

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Department of Mathematics
Faculty of Science
Benha University

Page of Title

Name: Mahmoud Mohamed Mokhtar

Degree: Doctor of Philosophy in Pure Mathematics

Department: Mathematics

Faculty: Science

University: Ain Shams

Graduation Date: 2002-faculty of science-Helwan University

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Abstract

This thesis is a contribution to numerical studies on systems of ordinary differential equations, fractional differential equations, integrodifferential equations and partial differential equations using different types of Chebyshev wavelets jointly with the spectral tau and collocation methods. Numerical tests are provided and the obtained result was compared with the published data in the literature moreover convergence analysis, error estimate and stability were discussed in details whenever it is possible. A detailed introduction to spectral methods, wavelets, and fractional calculus are given. The system of integrodifferential equations was solved using second kind Chebyshev wavelets. Coupled system of fractional differential equations was solved using third kind Chebyshev wavelets. The SIRC and Prey-Predator models were solved using first kind Chebyshev wavelets. Finally, the telegraph equation, KdV and Burger time fractional differential equations were solved using four different kinds of Chebyshev wavelets. All computations in this thesis were performed using Mathematica 9.

Keywords: Chebyshev Wavelets; Spectral Methods; Error Analysis; Fractional Differential Equations; SIRC Model; Telegraph Equation

Supervisors

1. Prof. Dr. Nasser H. Sweilam
2. Dr. I. K. Youssef
3. Dr. A. M. Nagy.

Signature

Summary

Summary The main aim of this thesis is to develop and implement spectral wavelets numerical algorithms for solving systems of ordinary/fractional differential, integrodifferential linear/nonlinear equations in addition to some applications in physics and biology, whenever possible we discuss in details the convergence of the presented techniques, also we compare our results with other existing techniques in literature.

The thesis is organized as follows:

♦ **Chapter one**:
We give a detailed introduction to numerical methods for solving differential equations, orthogonal polynomials and wavelets and some mathematical tools are given.

♦ **Chapter two**:
A new numerical scheme is presented to solve systems of integrodifferential equations are discussed. The derivation of this the scheme is essentially based on constructing the shifted second kind Chebyshev wavelets collocations methods. One of the main advantages of the presented scheme is its availability for application on both linear and nonlinear systems of integrodifferential equations. Another advantage of the developed scheme is that high accurate approximate solutions are achieved using a few number of the second kind Chebyshev wavelets. The obtained numerical results are comparing favourably with the analytical ones.

♦ **Chapter three**:
A numerical scheme is presented to solve a system of fractional differential equation is discussed. The derivation of this scheme is essentially based on constructing the shifted third kind Chebyshev wavelets collocations methods. One of the main

advantages of the presented scheme is its availability for application on both linear and nonlinear systems of fractional differential equations. Another advantage of the developed scheme is that high accurate approximate solutions are achieved using a few number of the third kind Chebyshev wavelets. The obtained numerical results are comparing favourably with the analytical ones.

⧫ **Chapter four**:

This chapter is devoted to implementing a Chebyshev wavelets method for obtaining the numerical solution for one of the well-known fractional SIRC, Prey-Predator models. In the SIRC case study, we concerned to discuss and study the effect of the rate of progression from infective to recovered per one year to the population density functions for various fractional Brownian motions and also for standard motion $N = 1$. We compared the obtained solutions with those obtained using 4^{th} Runge-Kutta method. From this comparison, we can conclude that the obtained numerical solution using the suggested wavelets method is in complete agreement with the numerical solution using RK4 method.

⧫ **Chapter five**:

In this chapter, four algorithms for obtaining numerical spectral wavelets solutions for Telegraph Equation and KdV, Burger type equation were analyzed and discussed. Chebyshev polynomials of four kinds are used. One of the advantages of the developed algorithms is high accurate approximate solutions are achieved using a few number of the Chebyshev wavelets. The obtained numerical results are comparing favourably with the analytical ones.

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