RELIABILITY OF SLENDER REINFORCED CONCRETE COLUMN

By

Inas Mohamed Saleh Abdel-Hasib

A thesis submitted to the

Faculty of Engineering at Cairo University

In partial Fulfillment of the Requirements of the Degree of

MASTER OF SCIENCE

IN

STRUCTURAL ENGINEERING

FACULTY OF ENGINEERING, CAIRO UNIVERSITY

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Title of Thesis:

Reliability of slender reinforced concrete column

Key Words:

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Summary:

In this thesis an investigation on reliability of slender reinforcement concrete columns using risk analysis program and Monte Carlo Simulation method is excuted . since most of the variables involved in column design (material properties, geometric characteristics, loads) are random, probabilistic methods are used in the analysis. The effect of the concrete compressive strength, amount of longitudinal steel, load eccentricity, and slenderness ratio on the column reliability are investigated. It was found that all these factors have a considerable impact on the response of slender reinforcement concrete columns.

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Eng: Inas Mahmed Saleh (2016)

ABSTRACT

The main objective of structural design is to ensure safety and functional performance requirements of a structural system for its target reliability levels.

The actual strength of a slender reinforced concrete column is different from the nominal value calculated according to Egyptian code 2007.

In this study, the reliability index for the reinforcement concrete column with rectangular cross section is studied. The variable parameters studied include the loads, the concrete compressive strength, the steel yield strength , the cross-section dimensions of concrete, the reinforcement ratio, and the location of steel placement.

Risk Analysis program was used to perform the analytical study. The effect of different variables on the reliability of reinforced concrete slender column was studied and presented in the form of tables and curves.

Chapter 1: Introduction

1.1 General

The importance of reliability theory in the area of civil engineering has received greater attention during the last two decades. This is due to the importance of studying the uncertainties of the existing loads, reinforced concrete material property strength and the member dimension. As a result, the nominal reinforced concrete material property strength and nominal existing loads computed by the engineering designer are different from the actual values. Besides, there are many uncertainties in the models used for determination of effects of loads and resistances of structures. The design of structural members must achieve the necessary safety and serviceability condition.

1.2 Reliability analysis

The reliability of structure is its ability to achieve its design purpose during a specified time. the reliability index is equivalent to the probability of safety . There are many sources of uncertainties existing in structural design such as loads , material property strength, concrete dimension, reinforced steel area and error in the model . there are random variables where any random variable is defined by its probability distribution function

1.2.1 Definition of failure

The term “failure” does not necessarily mean totally structure failure but it may mean that the structure cannot perform its purpose.
The concept of a "limit state" is used to define failure, this is because the limit state is the limit between safe and failure regions.

The safety of a structure was calculated using probability of failure. The probability of failure, P_f , can be derived by considering the probability of distribution of the random variables resistance “R” and load “U”
$g(R,U) = R-U$, the structure “fails” when the load exceeds the structural resistance.

The reliability index β, is usually used to measure structural safety. It is defined as the ratio of the mean to standard deviation.
The relation between the reliability index and probability of failure (p_f) for the case of normal distributed safety margin is defined as
$P_f = \varphi(-\beta)$
Where φ is the cumulative standard normal distribution function

1.2.2 Modes of failure

This undesired performance can occur by many modes of failure: exceeding load-carrying capacity for shear or bending moment, deflection more than allowable limits, buckling, cracking, corrosion and erosion.

1.3 Random variable

The random variables are considered as the parameters which represent all types of uncertainties that are included in input data to the considered model and has a probability distribution function.

1.4 Monte Carlo simulation method

Monte Carlo simulation method is a used technique to generate series of random variables which are defined by its probability distribution function and then by its basic statistical properties such as the mean and median values, the coefficient of variation , the standard of deviation, symmetry and skewness.

1.5 Reinforced concrete

Reinforced concrete is a composite material consisting of concrete and steel reinforcement bars. The concrete resists the compressive stresses and steel reinforcement resists tension stresses

Reinforced concrete is an economic, very strong material, easy to work and durable. It is commonly used for structures such as footings, structural elements, dams, piers etc.

1.5.1 Column

Column is the structural element that carries the compression forces from the different load type such as (dead load, live load, walls load, floor cover loads) of the structure above it. For the purpose of lateral loads column are designed to resist lateral forces. The columns are divided into two type: short columns and long columns.

1.5.1.1 Short Columns

The short columns strength is governed by the geometry of the cross section and the strength of the material. According to the Egyptian code for design and construction of concrete structures (2007), the ultimate short column load capacity

$(p_{ult}) = 0.35*A_{concrete}*f_{cu}+0.67*A_{steel}*f_y$ -------------------------- (1-1)

where; $A_{concrete}$ = area of concrete cross section
A_{steel} = area of reinforced steel
f_{cu} = concrete compressive strength
f_y = steel yield strength

1.5.1.2 Long columns

The long columns strength is governed by the geometry instability. The column is said to be long if its slenderness ratio is more than the values indicated in table (3-2) according to the Egyptian code for design and construction of concrete structures (2007). The long column that has slenderness ratio great than these values will collapse under a small compression load. The basic information on the behavior of long columns was determined using a mathematical analysis by Euler.

The Euler buckling load for an axially loaded pin ended column is given by:

$$P_E = \frac{\pi^2 EI}{L^2}$$

P_E = the Euler buckling load
E = Young’s modulus for the material
I = the least second moment of area of the section
L = the length of the strut between the pinned ends

1.6 Thesis objective

There is a lake of study on the reliability index of slender reinforced concrete columns. The objective of this thesis is to introduce a Risk analysis technique via the Risk analysis program model to identify and assess the parameters that affect the reliability index of slender columns. The reliability index of slender reinforced concrete column was designed according to the Egyptian code for design and construction of concrete structures (2007).

1.7 Thesis organization

The presented thesis is divided into eight chapters:

Chapter1, introduction.

Chapter 2, presents the reliability concept

Chapter 3, presents the limit state equations and the performance functions of slender reinforced concrete columns. it also shows the different failure modes of slender concrete column.

Chapter 4, describes the variables in reinforced concrete members and the chosen probability distribution function for each parameter of random variables.

Chapter 5, summarizes the literature review of the available previous pertinent research work in the field of reliability analysis and its common applications.

Chapter 6, includes the parametric study carried out to calculate reliability index of slender reinforced concrete column designed according to the Egyptian code for design and construction of concrete structures (2007) using risk analysis program.

Chapter 7, contains the analysis and discussion of the obtained analytical results and the summary of the results in tables and figures. The conclusions and the related future work, are demonstrated.

Chapter 8, a summary for the thesis and the conclusions are demonstrated.

Chapter 2: Reliability concept

2.1 Uncertainties in the Building Progress

During the different stages of the establishment of the structure there are many uncertainties in the design phase, construction or operation phase. These uncertainties belong to natural causes or human causes.

Natural causes of uncertainty are attributed to the unpredictability of existing loads such as dead load, live load, wind load, earthquake load, snow load, etc. Another source of natural uncertainty is the mechanical properties of the materials used in the construction of building. Human causes of uncertainty are attributed to approximation during design stage , miscalculation and lack of experience. During the operation stage the structure can be subjected to overloading due to change in the purpose of the use of structure (residential, administrative, industrial or commercial etc.) or due to lack of maintenance.

Generally, the uncertainties in structural design arise from many main factors: uncertainties due to the random nature of loads and resistance , the variation of load and resistance that is inherent in the quantity being considered, examples include a natural variation of wind pressure, earthquake, live load, and material properties. Statistical variation factor, this factor represents uncertainty arising from estimating parameters based on a limited sample size. In most situations, the natural variation (physical variation factor) is unknown and it is quantified by examining limited sample data. Therefore, the large the sample size, the smaller the uncertainty described by the statistical variation factor. Model variation factor: this factor represents the uncertainty due to simplifying assumptions, unknown boundary conditions, and unknown effects of other variables. It can be considered as a ratio of the actual strength (test result) and strength predicted using the model.

2.2 Definition of failure and safety index

Existing loads, materials strengths and properties and structural cross section dimensions are the main random variables during the design stage. Partial safety factors are used to increase loads and to decrease steel strengths and concrete strengths . For both of steel and concrete , the partial factors are used to cover the difference in the nominal cross section dimensions and the difference in the results between strength obtained from test specimens and the real strength in the field.

The use of partial safety factors, although convenient, is not sufficient to determine the safety level obtained in the design. In fact, safety depends on the structural response due to the actions and this involves interdependence among all random variables. Consider a large number of members, each is designed to have the same nominal resistance and assumed to be subjected to the same specified service load effect. Since loads are variable, the probability density of life-time maximum load effect U for all the members is similar to the plotted along the vertical axis in Figure (2-1) [50]. Similarity, due to variations in constituent material strengths, geometry, and design simplifications, the probability density of resistance R for all the members is similar to that plotted along the horizontal axis. The 45° a multiple of standard deviation by combinations of U and R that fall above this line represent the failure condition R< U.

A function Y= R/U representing to load effect can be obtained with a mean value of $\bar{Y}$ and a standard deviation of σy. The shaded area represents the failure condition Y (or R/U) < 1.0. The generalized reliability index, β can be considered as a quantitative measure for the safety. In general, the term β can be defined based on the probability of the occurrence of failures. the safety index β is equivalent to the inverse variation coefficient of the limit state function:

$\beta = \frac{(\bar{Y} - 1.0)}{\sigma y}$,

and is taken as a measure of structural reliability.

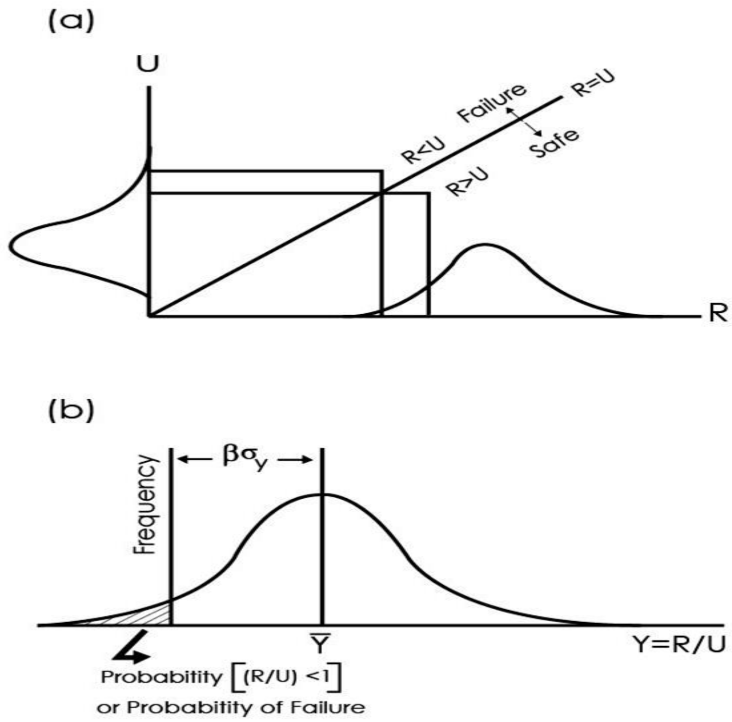

Figure 2-1: Definition of failure, and safety index β [50]

2.3 probability density functions [48]

There are several probability density functions which are used frequently in structural reliability. A probability density function that x will be occur between two intervals on the axis of real numbers, xa and xb, is

$$\Pr (xa \leq x \leq xb) = \int_{xa}^{xb} p(x)dx \qquad (2\text{-}1)$$

A probability density function must satisfy certain conditions:

1- The probability density function is always equal to or greater than zero

$$P (x) \geq 0.$$

2- The area under the probability density function is unity

$$\int_{-\infty}^{+\infty} p(x)\, dx = 1 \qquad (2\text{-}2)$$

3- A probability distribution function is the integral of a probability density function. The probability distribution function is defined as follows:

$$P(x_a) = \int_{-\infty}^{xa} p(x)dx = pr\,[\,x \le \text{xa}\,] \qquad (2\text{-}3)$$

And p(xa) is the probability that the random variable x will have a value equal to or less than xa.

2.3.1 Mathematical forms of the common probability density functions

2.3.1.1 Uniform two parameters:

$$P(x) = \left(\frac{1}{b-a}\right) \qquad a \le x \le b \qquad (2\text{-}4)$$

$$= 0 \qquad \text{other wise}$$

figure (2.2) shows its general shape.

The mean of x is as follows;

$$\bar{x} = \int_{-\infty}^{+\infty} x\, p(x)\, dx = \left(\frac{a+b}{2}\right) \qquad (2\text{-}5)$$

and its variance is as follows;

$$\sigma^2_x = \int_{-\infty}^{+\infty} \left(x - \bar{x}\right)^2 p(x)\, dx = \left(\frac{b-a}{12}\right)^2 \qquad (2\text{-}6)$$

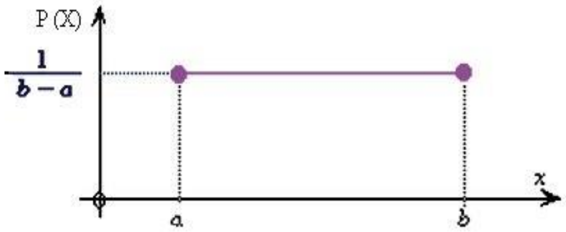

Figure 2-2: Uniform probability density function

2.3.1.2 Normal two parameters function:

$$P(x) = \frac{1}{a\sqrt{2\pi}} \exp\left(-0.5\right)\left(\left(\frac{x-b}{a}\right)\right)^2 \qquad (2\text{-}7)$$

$$-\infty \le x \le +\infty$$
$$a > 0\,,\; b > -\infty$$

figure (2-3) shows its general shape.
The mean of x is as follows;

$$\bar{x} = b \qquad (2\text{-}8)$$

and its variance is as follows;

$$\sigma^2 = a^2 \qquad (2\text{-}9)$$

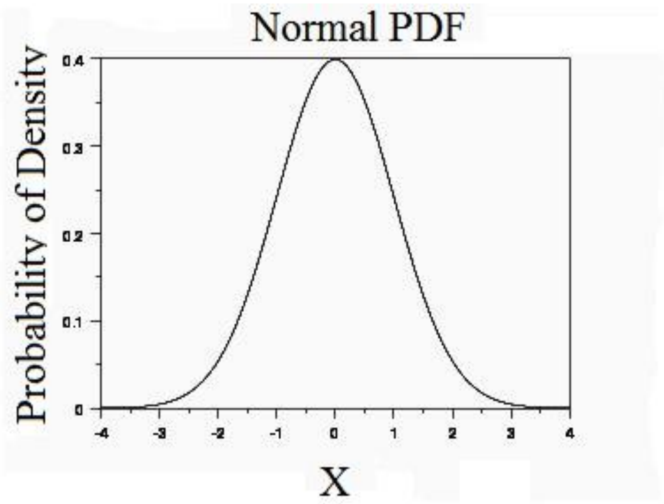

Figure 2-3: Normal probability density function(PDF)

2.3.1.3 Log normal two parameters function:

$$P(x) = \frac{1}{ax\sqrt{2\pi}} \exp\left(-0.5\left(\left(\frac{\ln x - b}{a}\right)^2\right)\right) \qquad (2\text{-}10)$$

$0 \le x \le +\infty$

$a > 0, \qquad b > -\infty$

The mean of x is as follows;

$$\bar{x} = \exp\left(b + \frac{a2}{2}\right) \qquad (2\text{-}11)$$

and its variance is as follows;

$$\sigma^2_x = (\exp(2b + a^2))(\exp(a^2) - 1) \qquad (2\text{-}12)$$

2.3.1.4 Log normal three parameters function:

$$P(x) = \frac{1}{(x-c)\,a\sqrt{2\pi}} \exp\left(-0.5\left(\left(\frac{\ln(x-c) - b}{a}\right)^2\right)\right) \qquad (2\text{-}13)$$

$c \le x \le +\infty$

$a > 0, \qquad b < \infty$

$C > -\infty$

The mean of x is as follows;

$$\bar{x} = c + \exp\left(b + \frac{a2}{2} \right) \qquad (2\text{-}14)$$

and its variance is as follows;

$$\sigma^2_x = (\exp(2b + a^2))(\exp(a^2) - 1) \qquad (2\text{-}15)$$

2.3.1.5 Gamma two parameters function:

$$P(x) = \frac{1}{a\,\Gamma(b)} \left(\frac{x}{a}\right)^{b-1} \exp\left(-\frac{x}{a}\right) \qquad (2\text{-}16)$$

Where $\Gamma(b)$ *is the gamma function*

$$\Gamma(b) = \int_{-\infty}^{+\infty} y^{b-1} \exp^{(-y)}\, dy \qquad (2\text{-}17)$$

The mean of x is as follows;

$$\bar{x} = ab \qquad (2\text{-}18)$$

and its variance is as follows;

$$\sigma^2_x = a^2 b \qquad (2\text{-}19)$$

2.3.1.6 Gamma three parameters function:

$$P(x) = \frac{1}{a\,\Gamma(b)} \left(\frac{x-c}{a}\right)^{b-1} \exp\left(-\left(\frac{x-c}{a}\right)\right) \qquad (2\text{-}20)$$

$$X > c,\ a > 0,\ b > 0$$

The mean of x is as follows;

$$\bar{x} = c + ab \qquad (2\text{-}21)$$

and its variance is as follows;

$$\sigma^2_x = a^2 b \qquad (2\text{-}22)$$

2.3.1.7 Exponential two parameters function:

The exponential probability distribution function. is a special case of the three-parameter gamma probability distribution function ; if b = 1 , then

$$p(x) = \frac{1}{a} \exp\left(-\left(\frac{x-c}{a}\right)\right) \qquad (2\text{-}23)$$

$$x > c,\ a > 0$$

2.3.1.8 Beta four parameters function a ,b, c and d :

$$p(x) = \left(\frac{1}{b-a}\right) [\beta(a,b)]^{-1} \left(\frac{x-a}{b-a}\right)^{c-1} \left[1 - \left(\frac{x-a}{b-a}\right)\right]^{d-1} \qquad (2\text{-}24)$$

$$a \le x \le b$$
$$0 \le a \le b$$
$$c > 0,\ d > 0$$

where;

$$\beta(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

The mean of x is as follows;

$$\bar{x} = a + (b-a)\left(\frac{c}{c+d}\right) \qquad (2\text{-}25)$$

and its variance is as follows;

$$\sigma^2_x = (b - a)^2 \left[\frac{cd}{(c+d)(c+d+1)} \right] \qquad (2\text{-}26)$$

Review of commonly used probability distributions for design variables shows that the normal distribution provides a reasonable representation of the variability in these variables, and simplifies probability calculations. Although the relative ease of probability calculations in the case of normally distributed variables, but for better decision making, it is important to study the probabilistic nature of loads and material strength to find out the most probabilistic model.

Reliability evaluation of a structure requires the identification of the basic variables by which the uncertainties related to the reliability of the structure can be described. These variables are material strengths, external loads and geometrical quantities. Some of these variables are dealt with as random variables which may be described by their statistical parameters such as their mean values, standard deviations and distribution functions.

2.4 Methods for estimating the statistical properties of column's capacity.

An important area of statistics is concerned with the estimating of the parameters of a probabilistic model. Three methods are presented for estimating the statistical properties of a probabilistic model knowing its parameter's, mean and variance.

The first method uses the linear matrix transformation and, the second method uses the Taylor's series expansion, while the third method involves the use of a computer, and is called the Monte Carlo analysis.

2.4.1 Linear matrix transformation [48]

Matrix equations often relate one set of variables to another by a linear matrix transformation.

$$\{ y \} = [c] \{ x \} \qquad (2\text{-}27)$$

Where

[c] = (m*n) matrix of constants
{ x } = (n*1) vector of random variables
{ y } = (m*1) vector of random variables

Expanding above equation, gives the following quadratic form;

$$Y_j = \sum_{k=1}^{n} c_{jk}\, x_k \qquad j= 1,2,\ldots\ldots,m \qquad (2\text{-}28)$$

Taking the mean value of each side of this equation :

$$\bar{Y}_j = \sum_{k=1}^{n} c_{jk}\, \bar{x}_k \qquad j= 1,2,\ldots\ldots,m \qquad (2\text{-}29)$$

The covariance matrix of the { Y} set of random variables, can be expressed in terms of the covariance matrix of the{ x } set of random variables. Consider two random variables Y_j and Y_k.

$$Y_j = \sum_{i=1}^{n} c_{ji}\, x_i \qquad (2\text{-}30)$$

and

$$Y_k = \sum_{h=1}^{n} c_{kh}\, x_h \qquad (2\text{-}31)$$