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Institute of Statistical Studies and Research  
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# **An Intelligent Search Technique For Solving Multi-Objective Optimization Problems**

By

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## Certification

I certify that this work has not been accepted in substance of any academic degree and is not being concurrently submitted in candidature of any other degree.

Any portion of this thesis in which I am indebted to other sources are mentioned and explicit references are given.

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Signature: .....

# Dedication

*My mother & father*

They were here at the beginning but gone at the end

*My brother*

He supports me from the beginning till the end

*Cairo Oil & Soap Company*

My home where my family exists

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## List of Abbreviations

ACO	Ant Colony Optimization
AIS	Artificial Immune Systems
CA	Cellular Automata
DM	Decision Maker
EAs	Evolutionary Algorithms
GAs	Genetic Algorithms
MADA	Multi-Attribute Decision Analysis
MCDM	Multiple Criteria Decision Making
MOCO	Multi-Objective Combinatorial Optimization
MOEA	Multi-Objective Evolutionary Algorithm
MOP	Multi-objective Optimization Problem
MOPSO	Multi-Objective Particle Swarm Optimization
NE	Net Energy
$NE_g$	Net Energy required for Growth
$NE_m$	Net Energy required for Maintenance
PSO	Particle Swarm Optimization
RD	Relative Divergence
RD-MOPSO	Relative Divergence Multi-Objective Particle Swarm approach
SA	Simulated Annealing
SI	Swarm Intelligence

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## Summary

In this thesis work, we introduce a novel scheme to maintain diversity among nondominated solutions found in the external archive, and captured during the search in multi-objective problems. Besides we redefine diversity of solutions into two factors showing that the Relative Divergence (RD) scheme satisfies both factors while the previous related work did not, we also show the advantages of the new scheme over the previous related work.

This scheme is incorporated into particle swarm optimization algorithm resulted in Relative Divergence Multi-Objective Particle Swarm Optimization approach (RD-MOPSO) proposed in this work, we use the new scheme for leaders' selection problem which is a key component when designing a MOPSO approach. The new scheme attains better diversification of selected solutions to be leaders which will be reflected finally on the quality of solutions produced by the algorithm (better diversification). The thesis contains six chapters, and it is organized as following:

Chapter 1: This chapter gives an introduction, showing the outline of the thesis, motivation and contributions of this work.

Chapter 2: This chapter is devoted for basic concepts, this chapter presents Multi-Objective Optimization, Evolutionary Algorithms concepts in brief, and then Multi-Objective Evolutionary Algorithms field is introduced showing its backgrounds. In this chapter an overview on the main features of Swarm Intelligence paradigms in general, focusing on Particle Swarm Optimization field, showing its main principles, terms used in PSO field, a comprehensive survey on the applications of PSO is presented. In the last section of this chapter Multi-Objective Particle Swarm Optimization field is introduced showing the modifications done to original PSO algorithm, challenges and issues aroused when introducing a new proposal in MOPSO field.

Chapter 3: This chapter is made for the previous related work done in Multi-Objective Particle Swarm Optimization field; a complete survey of the state-of-the-art is presented encapsulating 40 proposals to be listed in one table in the end of the chapter.

Chapter 4: In this chapter an important survey is done for the most outstanding schemes presented for leaders' selection problem, analyzing their ideas and showing their advantages, focusing on their drawbacks and limitations.

Chapter 5: In this chapter a novel scheme (RD) is introduced to maintain diversity among nondominated solutions found in the external archive during the process of leaders' selection, also it is shown the advantages of the new scheme over the previous related work. The new scheme is incorporated into particle swarm optimization technique resulted in a new approach (RD-MOPSO), test functions and metrics is done for validation, comparing our approach to six state-of-the-art algorithms and introducing results. In the end of the chapter we introduce a case study found in Cairo Oil and Soap Company, also solution done by our approach is presented.

Chapter 6: This chapter is made for fundamental results and conclusions.

# **Chapter 1**

## **Introduction**

### **1.1 Background**

All areas of human interaction with its environment involve decision situations, decision making should ideally be based on complete knowledge of the alternatives at hand as well as their consequences. Daily we face hundreds of situations to take decisions about, in which it might be so hard (impossible in some cases) to have exact predictions of their consequences or final results, due to the complex nature of these systems we usually have to rely on models, which provide approximations to reality [36]. Here, systems analysis plays an important role, since only a well informed decision maker is in a position to take decisions. In systems analysis, three interrelated activities can be distinguished based on different points of interest:

***Modeling:*** What are the mechanisms that produce a certain behavior or output on a given input, and how can they be described?

***Simulation:*** What output is produced by the system for a given input?

***Optimization:*** What input needs to be provided to the system in order to receive a desired or optimal output?

All three areas are represented by established scientific disciplines with their own methodologies and approaches. This thesis focuses on optimization, i.e., the search for optimal solutions among a set of alternatives.

In general, any optimization problem  $P$  can be described as a triple  $(S, \Omega, f)$ , where:

1.  $S$  is the search space defined over a finite set of decision variables  $X_i, i = 1, \dots, n$ . In the case where these variables have discrete domains we deal with discrete optimization (or combinatorial optimization), and in the case of continuous domains  $P$  is called a continuous optimization problem. Mixed variable problems also exist.  $\Omega$  is a set of constraints among the variables;
2.  $f : S \rightarrow R$  is the objective function that assigns a cost value to each element (or solution) of  $S$ .

The goal is to find a solution  $s \in S$  such that  $f(s) \leq f(s'), \forall s' \in S$  (in case we want to minimize the objective function), or  $f(s) \geq f(s'), \forall s' \in S$  (in case the objective function must be maximized). In real-life problems the goal is often to optimize several objective functions at the same time. This form of optimization is named Multi-Objective Optimization which we are interested in this thesis work [4].

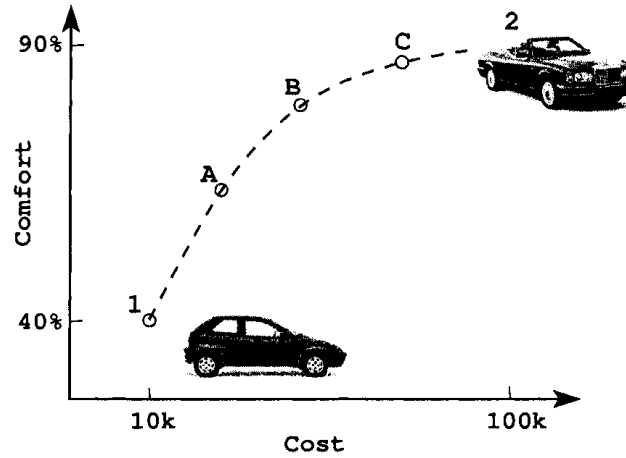
Due to the practical importance of optimization problems, many algorithms to tackle them have been developed in both areas the Single and Multi-Objective Optimization.

## 1.2 Why Multi-Objective Optimization?

Decision situations often involve multiple criteria or objectives. In many cases, objectives are *incommensurable*, meaning they are not comparable with respect to magnitude and value, and *conflicting*, meaning that the different objectives cannot be arbitrarily improved without decreasing the value of another. This result in trade-offs between the objectives [36].

Let us consider the decision-making involved in buying a car presented by Deb in [19] which helps us in understanding the idea of Multi-Objective Optimization through the

following illustrative example. Cars are available at prices ranging from a few thousand to few hundred thousand dollars. Let us take two extreme hypothetical cars, i.e. one costing about ten thousand dollars (solution 1) and another costing about a hundred thousand dollars (solution 2), as shown in Figure 1.



**Figure 1. Hypothetical trade-off solutions for a car-buying decision-making problem [19].**

If the cost is the only objective of this decision-making process, the optimal choice is solution 1. If this were the only objective to all buyers, we would have seen only one type of car (solution 1) on the road and no car manufacturer would have produced any expensive cars. Fortunately, this decision-making process is not a single-objective one. It is expected that an inexpensive car is likely to be less comfortable. The figure indicates that the cheapest car has a hypothetical comfort level of 40%. To rich buyers for whom comfort is the only objective of this decision-making, the choice is solution 2 (with a hypothetical maximum comfort level of 90%, as shown in figure 1).

This so-called bi-objective optimization problem need not be considered as the two independent optimization problems, the results of which are the two extreme solutions discussed above (solutions 1 and 2). Between these two extreme solutions, there exist many other solutions, where a trade-off between cost and comfort exists. A number of such solutions (solutions A, B, and C) with differing costs and comfort levels are also shown in the figure. Thus, between any two such solutions, one is better in terms of one objective, but this betterment comes only from a sacrifice on the other objective. In this sense, all