TREATMENT SOME OF THE PROBABILISTIC INVENTORY SYSTEMS

A THESIS

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Chapter I

Fundamentals of Inventory system

1.1 Introduction

The analysis of inventory system has appeared since more than ninety years ago in literature. The earliest derivation of what often called the simple lot size formula was obtained by Ford Harris [28] of Westinghouse Corporation in 1915. The first full length introductory book to deal with inventory problems was that of Raymond [43], it contains no theory or derivations but only attempts to explain how various extents of the simple lot size model that can be used in practice. The paper by economists Arrow, et. al [3] was one of the first to provide a rigorous mathematical analysis of a simple type of inventory model.

The inventory models are necessarily needed for developed countries due to the supply and demand processes differ in the rates at which they respectively provide or require stock. The inventory control is defined as the science based on the technique of controlling the amount of stocks held within a business to meet economically the demands placed upon that business.

There are many reasons that organizations should maintain inventories of goods. The fundamental reason is either physically impossible or economically unsure to have goods arrive in a given system precisely when demands for them occur without inventories, customers would have to wait until their orders were filled from a source or were manufactured. In general, however, customers are not allowed to wait for long periods of time. For this reason, the carrying cost of inventories is necessary to almost all organizations that supply physical goods or commodities to customers. Another reason for maintaining inventories is that sales and profits can be increased if one has an inventory of goods to display to customers. Two fundamental questions must be answered in controlling the inventory of any physical goods, are when to place the order and how much to order for procurement.

Dvoretzky, et. al [10,11] had taken an interest in inventory control models. They were not been concerned with immediate practical applications; instead they were interest in the models because of their mathematical properties and economic interpretations.

Any inventory system must be one of the following four systems:

- Single-item, single-source (SISS)
- Single-item, multi-source (SIMS)
- Multi-item, single-source (MISS)
- Multi-item, multi-source (MIMS)

In this thesis, we are dealing with probabilistic SISS system and probabilistic MIMS system. The basic assumption of the SISS concept is that replenishment of the item can be made from one source only. In the MIMS the replenishment for the aggregate of items could be done from one of several possible sources.

Most literatures deal with probabilistic inventory models assume that the demand rate is probabilistic since the probability distribution of the future demand rate, rather than the exact value of demand rate itself, is known. The probability distribution of future demand is usually determined from the data collected from past experience. In such situations we choose policies that minimize the expected costs rather than the actual costs.

Most of the classical probabilistic inventory models assume that the unit purchase price, order cost and the holding cost are all constants and independent of the order quantity Q. This assumption is not really in practice since the holding cost is always a function of the unit price of an item that varies with the order quantity. Similarly, the order cost may depend on the order quantity Q.

Richards [44] presented comments on the distribution of inventory position in a continuous review (S,s) inventory system. Goyal [23] discussed multi-product inventory situations with one restriction. Maloney [33] introduced the single period, multi-product inventory system with limited capacity, he also [34] studied the multi-product inventory system under the budget, warehouse space constraints.

Recently, Abou-El-Ata and Kotb [2] treated the deterministic multi-item EOQ inventory model with varying holding cost under two restrictions: a geometric programming approach. Also, Mousa [37] studied constrained deterministic multi-item inventory models using a Geometric programming approach. The Geometric programming approach enables them to get a closed form of the optimal solution. Fergany, et. al [16,17] deduced constrained probabilistic multi-item inventory models with varying holding cost. A more recent work by Fergany, et. al [18,19,20] investigate constrained probabilistic single-item and multi-item inventory models with varying units cost using the Geometric programming technique and Lagrange multiplier.

In this thesis, the Geometric programming approach is used to minimize the expected total cost of constrained MIMS buffer stock inventory system with zero lead time and varying order cost. Then, we deduced two types (backorders and lost sales) of continuous review inventory systems with varying order cost and the restriction on the expected holding cost when the lead time demand is a continuous random variable with known distribution and lead time is constant. Also, we derived the model that combines the previous backorder and lost sales models to obtain the model of mixture shortage. The Lagrange method is used to find the optimal order quantity and the optimal reorder point which minimize the relevant expected total cost. Finally, we discussed the mixture shortage model when the demand and the lead time are two random variables with known distribution while the distribution of the lead time demand is unknown.

1.2 Types of Inventories

There are five types of inventory, namely:

(i) Transportation inventories

(ii) Buffer stock inventories

(iii) Anticipation inventories

(iv) De coupling inventories

(v) Lot-size inventories (continuous review inventories)

In this thesis, we deal with two types of inventories defined as follows:

- **Continuous Review Inventories**

  Continuous review inventory is one such that a quantity Q is ordered each time the appropriate inventory level reaches the reorder point r. This policy is known under the short $\langle Q,r \rangle$ systems. Generally, the status of stock is always known in the continuous review system.

- **Buffer Stock Inventories**

  Buffer stocks are extra inventory maintained to meet the uncertainty of demand and supply. They are needed to cover the demand during the lead time in case actual demand exceeds expected demand or actual lead time exceeds expected lead time.

1.3 Inventory Costs

There are four kinds of inventory costs associated with keeping inventories of items. These are:

(i) Purchase (production or item or buying) cost

(ii) Ordering (or set-up or procurement) cost

(iii) Holding (or carrying) cost

(iv) Shortage (or stock out) cost

Purchase Cost

It refers to the cost associated with items whether it is manufactured or purchased usually denoted by C_p. It refers as the purchase cost if it is obtained