



Ain Shams University
University College for Women
(Arts, Science, and Education)
Department of Mathematics

A Theoretical Study on the Optical Properties of Low Dimensional Nanostructures

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FOR THE DEGREE OF
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(APPLIED MATHEMATICS)

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Thesis Supervisors:

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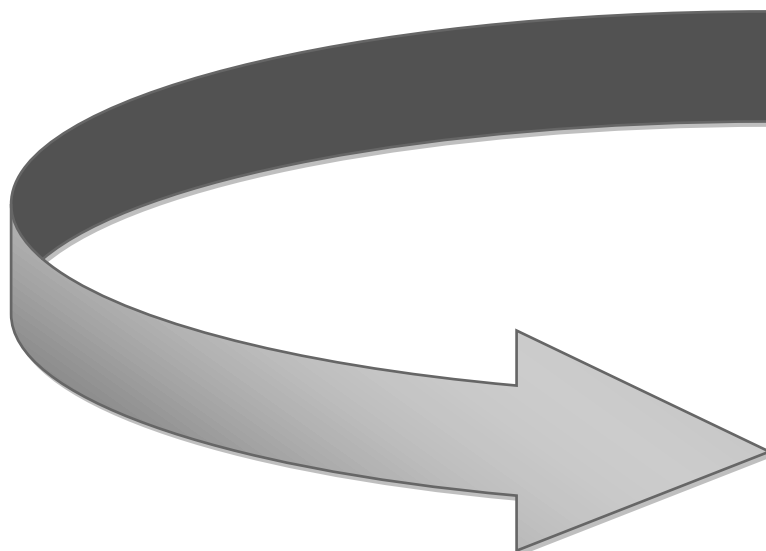
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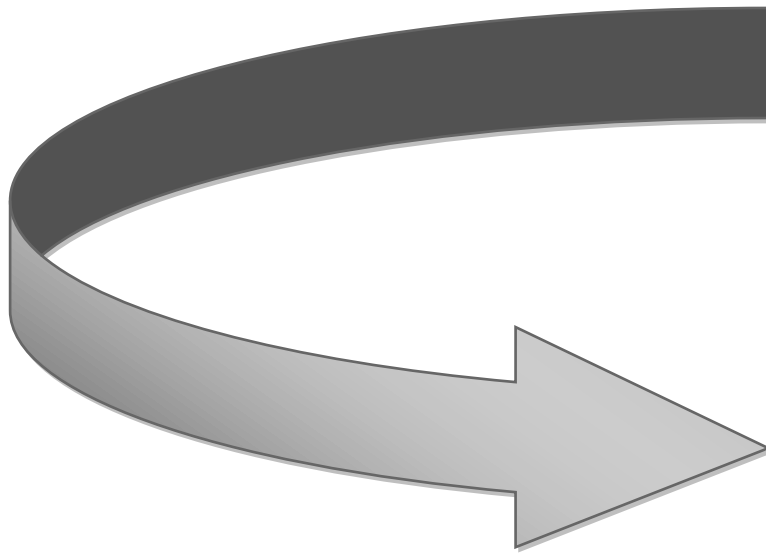
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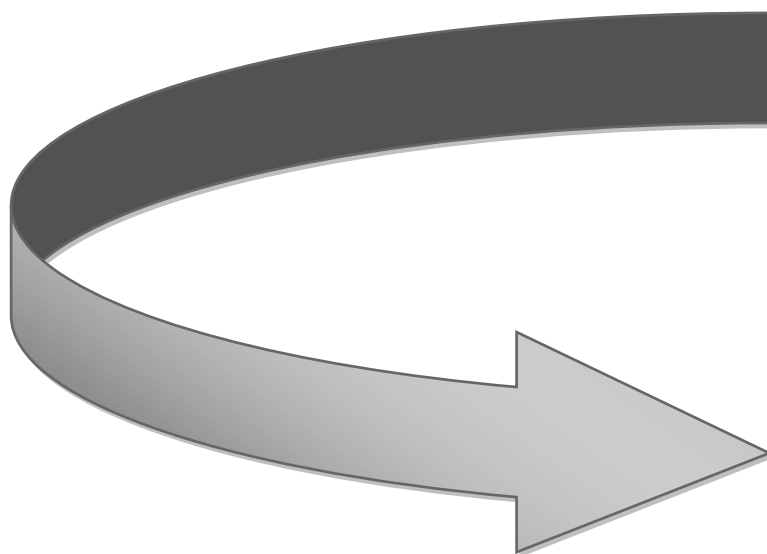
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ABSTRACT



Abstract

The linear, the third order nonlinear optical absorption and the third harmonic generation susceptibility have been investigated by using the density matrix technique. The confinement potential has been taken to be a combination of a parabolic and inverse squared part. The study is distinct from previous studies by including the effect of a central or off-central hydrogenic impurity located in different positions. In the absence of impurity exact analytical solutions in terms of the associated Laguerre polynomials have been derived. However, in the presence of impurity the resulting Schrödinger equation has been solved by applying the variational method.

In the case of a two dimensional disk shaped quantum dot in the presence of a magnetic field the linear and nonlinear optical absorption coefficients have been calculated. The presence of an impurity causes a huge increase in the square of the main transition matrix $|M_{21}|^2$ and in the absorption coefficients.

In the case of semi-parabolic and semi-inverse squared quantum well the third harmonic generation (THG) has been investigated. Its imaginary part has also been displayed. This part yields valuable information concerning the third order absorption coefficient.

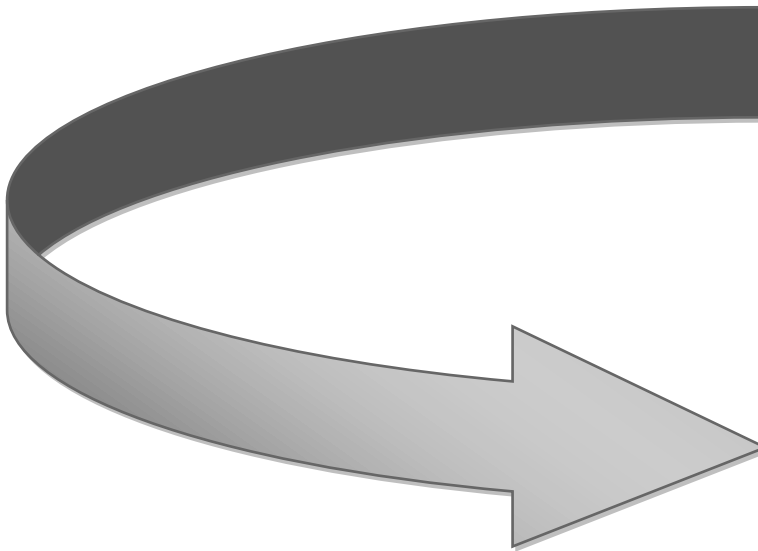
The results obtained in both cases have been compared with those obtained in the absence of impurity for each case.

The main results of chapter three have been published in I.F.I.Mikhail, A.M.Shafee, Physica B **507** (2017) 142.

Also, the main results of chapter four have been prepared for publication and will be submitted to an international specialized journal in the near future.

CHAPTER (1)

INTRODUCTION



CHAPTER ONE

INTRODUCTION

1.1 Review of the nonlinear optical properties in low dimensional structures:

In the past few years the nonlinear optical properties of low dimensional structures have been speculated by many authors and along different directions. The confinement of quantum systems leads to a substantial increase of the nonlinear optical properties than in bulk materials.

The properties of low dimensional structures depend, in general, on many factors. The geometry of the system may change from two dimensions (superlattices, quantum wells, disks), one dimension (quantum wires) and zero dimension (quantum dots). The confining potential may be infinite squared, parabolic, Gaussian or other complicated forms. Also, the presence of impurities plays an essential role. They may be central or off-central, hydrogenic or nonhydrogenic, donor or acceptor. Moreover, the applied external fields give a major contribution in the calculation or measurement of any involved coefficient. The applied external fields are frequently taken, to be electric, magnetic or both. The directions of these fields are also essential in determining the required properties.

The most powerful technique applied in earlier work to derive suitable expressions for the linear and nonlinear optical coefficients is the density matrix operator method (Boyd [5]).

We start in the following by reviewing some of the related previous work in spherical quantum dots.

The linear and third order nonlinear absorption coefficients in a spherical quantum dot have been investigated by Yilmaz and Sahin [62] in the presence of a central impurity. The confining potential is taken to be infinite or squared with different constant height. In fact these are the most simpler forms of the confining potential. In order to obtain the required energy levels and wave functions in the presence of the central impurity they applied the variational method. Most of the expressions which they have obtained were derived earlier in the work of Montenegro and Merchancano [39]. Also, more accurate results can be inferred by using the exact solution of the Schrödinger equation in the presence of a central impurity obtained later by Mikhail and El Sayed [34]. Yilmaz and Sahin [62] concluded that the radius of the dot and the height of the confinement potential have a great effect on the linear and nonlinear absorption coefficients.

The linear and nonlinear optical properties of a spherical quantum dot in the presence of central impurity have also been studied by Özmen *et al* [40], Yakar *et al* [58], [59] and Cakir *et al* [6], [7]. They have mainly used a modified variational optimization approach that depended on expanding the wave function in terms of slater-type orbital functions with the coefficients as variational parameters. In all these references a squared finite confining potential has been considered except in Yakar *et al* [59] where a parabolic confining potential has been added.