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بالرسالة صفحات
لم ترد بالأصل

REGULARIZED TRACE FOR EIGEN-FUNCTIONS AND EIGEN-VALUES OF STURM-LIOUVILLE OPERATORS WITH DIFFERENT FORMS OF BOUNDARY CONDITIONS

A thesis

Submitted to the Department of Mathematics, Faculty of Science, Tanta University in the Partial Fulfilment of the Requirements for the Award of the Degree of Master of Science

In

{Pure Mathematics}

By

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تم عمل التصحيحات المطلوبة

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Beside the research work materialized in this thesis, the candidate has attended five postgraduate courses within two years including the following topics :

1- Abstract Algebra

2- Numerical Analysis

3- Partial Differential Equations.

4- Functional analysis.

5- German language.

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CONTENTS


PREFACE

PREFACE

It is well known that the summation of the diagonal elements in a square matrix is equal to the summation of the eigenvalues of linear operator in finite dimensional space. In other words, the trace of a matrix is equal to the spectral trace in n-dimensional space.

It is worth mentioning that this theorem is satisfied also in the case of unclear operators which are defined in Hillbert space [24]. Thus we might ask the following question : Is this theorem applicable in case of unbounded operators ? ; especially in the case of differential operators the trace of matrices and spectral trace are not exist. For this reason we define the so-called "Regularized trace".

The study of regular trace for differential operators plays an important role in several fields such as mathematical analysis, theoretical physics and quantum mechanics. We can use also the regular trace in the inverse spectral problems in functional analysis.

A good number of works has been devoted to the deduction of the formulae of regularized traces of differential operators such as L. M. Gelfand, B. M. Levitan [1], J. A. Charls, JR. Halberg and V. A. Kramer [2], V. B. Lidsky, V. A. Sadovnichii [3], [4], V. A-Sadovnichii, V. A. Lyubishkin [5], Y. Belabbaci [6], S. A. Saleh [7], H. A. Zedan [8], S. A. Saleh, M. A. Kassem [9], S. A. Saleh, R. M. Allam [10], V. A. Lyubishkin [11], A. S. Pechentsov [12]-[13], V. A. Sadovnichii, V. V. Dubrovskii [14], [16], D. Milinkovic [15] and others.

The concept of regularized sum for eigenfunction of differential operators are introduced by S. A. Saleh [18], S. A. Saleh R. M. Allam [19]. Also the proof of expansion theorem of eigenfunction for multi-point and integral conditions is given by S. A. Saleh, M. A. Kassem [20].

The main role of this thesis is calculating the regularized traces of eigenvalues and regularized sum of eigenfunctions of Sturm-Liouville operator under different conditions i.e., the differential equation

$$-y'' + q(x)y = z^2 y(x), \quad x \in [0, \pi]$$

under the conditions

$$\Gamma_i(y) = a_{i1}y(0) + a_{i2}y'(0) + a_{i3}y(\pi) + a_{i4}y'(\pi), \quad i = 1,2$$

The thesis consists of four chapters, in the first chapter, we introduce some fundamental definitions and theorems which considered the necessary background material for the other three chapters. Accordingly we wrote a short notes on the following

1- The Sturm-Liouville boundary value problem on a bounded interval.

2- Asymptotic formulas for solutions of Sturm-Liouville equation.

3- Asymptotic formulas for eigenvalues and trace formulas.

In chapter II, we obtained the regularized traces of eigenvalues and regularized sum of eigenfunctions if $J_{42} \neq 0$, where

$$J_{42} \neq a_{14}\, a_{22} - a_{24}\, a_{12},$$

Also we introduced some important examples which follow the case I ($J_{42} \neq 0$).

In chapter III, we proved some important theorems which give the formulas of regularized traces of eigenvalues in case II i.e., $J_{42} = 0$; $J_{14} + J_{32} \neq 0$, where

$$J_{14} = a_{11}\, a_{24} - a_{21}\, a_{14}, \quad J_{32} = a_{13}\, a_{22} - a_{23}\, a_{12}$$

Also we obtained the formula of regularized sum of eigenfunctions, some examples are solved according to case II.

In chapter IV, we obtained the regularized traces of eigenvalues in case III i.e., $J_{42} = J_{14} + J_{32} = 0$, $J_{13} \neq 0$,

where

$$J_{13} = a_{11}\, a_{23} - a_{21}\, a_{13}$$

Also some important examples are solved in case III, and the regularized sum of eigenfunctions for one problem is obtained.

The results which are obtained in chapters II, III, IV are considered the general form comparing the previous studies, furthermore, we introduce the concept of regularized sum of eigenfunction in every case.