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SUMMARY

Peristaltic motion is a form of fluid transport generated by a progressive wave of area contraction or expansion along the length of a distensible tube containing fluid. In physiology, peristalsis is used by the body to propel or mix the contents of a tube as in the transport of urine from kidney to the bladder, the movement of chyme in the gastrointestinal tract, vasomotion of the small blood vessels, the bile duct and other glandular ducts. Some worms use peristalsis as a means of locomotion. Roller and finger pumps using viscous fluids also operate on this principle. Peristalsis has been proposed as a mechanism for the transport of spermatozoa in vas deferens. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited.

In this thesis, we innovated and solved analytically some problem of peristaltic motion for non-Newtonian fluids with heat and mass transfer in the presence of external effects such as the magnetic field, chemical reaction, mixed diffusion etc., on the flow region and/or presence of endoscope inside concentric and eccentric tubes which have peristaltic motion under the following assumptions and conditions:

- Long wavelength approximation.
- Boussinesq approximation.
- Ostrach approximation.
- A slip velocity and temperature jump conditions.
- A shear stress jump conditions.

To understand the physiological processes that occur in the human body and analyze the physical mechanisms that underline them which have helped the engineering design and

construction of new medical devices and machinery.

The present thesis consists of seven chapters in addition to appendices and references section and arabic and english summaries. These chapters are outlined as follows:

Chapter 1

This chapter includes introduction about two parts which are closely related to the subjected of the thesis

Part I : Introduction to Fluid Mechanics

Part II : Applications in Biomechanics

Chapter 2:

In this chapter, we have analyzed an incompressible flow of electrically conducting biviscosity fluid through an axisymmetric non-uniform tube with a sinusoidal wave under the considerations of long wavelength and low Reynolds number. In our analysis we are taking into account the induced magnetic field. The analytic solution has been obtained from which the axial velocity, the axial induced magnetic field and the axial pressure gradient have been derived. The results for the pressure rise, frictional force per wavelength and the axial induced magnetic field have been computed numerically and the results were studied for various values of the physical parameters of interest, such as the magnetic Prandtl number, the magnetic field parameter, the Reynolds number Re , the upper limit apparent viscosity coefficient, and the time averaged mean flow rate.

The contents of this chapter have been published in: Physica A, vol. 383 (2007), pp. 253-266.

Chapter 3:

In this chapter, The effect of chemical reaction on peristaltic motion of an incompressible Jeffrey fluid through vertical porous media in the gap between concentric tubes with heat and mass transfer has been studied with particular reference to an endoscope. The inner tube is uniform while the outer tube is non-uniform tube and has a sinusoidal wave traveling down its wall. Long wavelength approximation (that is, the wavelength of the peristaltic wave is large in comparison with the radius of the tube) is used to linearise the governing equations. A perturbation solution is obtained, which satisfies the momentum, energy

and concentration equations for case in which the porosity parameters is small. Numerical results for the pressure rise and frictional force per wavelength as well as for the skin-friction, Nusselt number and Sherwood number are obtained. Several graphs of physical interest are displayed and discussed.

The contents of this chapter have been submitted for publication to: Communications in Nonlinear Science and Numerical Simulation (2007).

Chapter 4:

In this chapter, we investigated the effect of both of magnetic field and porous medium on non-Newtonian fluid by studying the unsteady flow of a compressible biviscosity fluid in a circular tube, in which the flow is induced by a wave travelling on the tube wall. Also, we evaluated the effect of the non-Newtonian (biviscosity) property of blood on the net flow rate of this fluid in arterioles. The important application of this problem is the flow of blood through arterioles subjected to a constant magnetic field is made the body gets more of the nutrients it needs from the blood. We have noticed that the net flow rate increases as the porosity parameter increases whereas it decreases with increasing the magnetic parameter. Furthermore, the fluid can be moved in the opposite direction to the propagation of the wave travelling on the tube wall.

The contents of this chapter have been published in: Bulletin of the Calcutta Mathematical Society, vol. 99 (2) (2007), pp. 123-136.

Chapter 5:

In real systems there is always a certain amount of slip, which, however, is hard to detect experimentally because of the required space resolution. In this chapter, we analyze the effect of slip boundary conditions on the dynamics of fluids by studying the peristaltic flow and heat transfer of an incompressible, electrically conducting Bingham non-Newtonian fluid in an eccentric uniform annulus in the presence of external uniform magnetic field. The viscous and Joule dissipations are taken into account. The temperature jump at the wall, ignored in recent investigations, is found to be essential importance in the heat transfer analysis. The inner tube is rigid and moving with a constant axial velocity, while the outer tube has a sinusoidal wave traveling down its wall. This analysis can model the gastric

juice motion in the small intestine when an endoscope is inserted through it. Under zero Reynolds number condition with the long wavelength approximation, the axial velocity and the stream function are obtained analytically. Numerical solution for the governing partial differential equation of energy is performed in order to analyze the temperature distribution. The effects of all parameters of the problem are numerically discussed and graphically explained.

The contents of this chapter have been submitted for publication to: Rheologica Acta (2007).

Chapter 6:

In this chapter, we investigated the problem of the unsteady mixed convection peristaltic mechanism. The flow includes a temperature-dependent viscosity with thermal diffusion and diffusion-thermo effects. The peristaltic flow is between two vertical walls, one of which is deformed in the shape of traveling transversal waves exactly like peristaltic pumping and the other of which is a parallel flat plate wall. The equations of momentum, energy, and concentration are subject to a set of appropriate boundary conditions by assuming that the solution consists of two parts: a mean part and a perturbed part. The solution of the perturbed part has been obtained by using the long-wave approximation. The mean part has been solved and coincides with the approximation of Ostrach. The mean part (zeroth order), the first order, and the total solution of the problem have been evaluated numerically for several sets of values of the parameters entering the problem. The skin friction, and the rate of heat and mass transfer at the walls are obtained and illustrated graphically.

The contents of this chapter have been accepted for publication in: Archive of Applied Mechanics (2008) (in press).

Chapter 7:

In this chapter, The peristaltic pumping of a biofluid consisting of two immiscible fluids of different viscosity, one occupying the core and the other the peripheral layers on either side, in a two-dimensional tube partially filled with a layer of a porous material is investigated. The core region is described by Eyring-Powell model and the peripheral region is taken to be electrically conducting Newtonian viscous fluid. The fluid in peripheral region is

permeated by an external uniform magnetic field imposed perpendicularly to the xy -plane on the assumption of a small magnetic Reynolds number in the presence of the effect of Hall currents. It serves as a model for the study of flow of chyme in gastrointestinal tract. The flow is examined in the wave frame of reference moving with the velocity of the wave. Brinkman extended Darcy equation is utilized to model the flow in porous layer. A shear stress jump boundary condition is used at the interface. The analytic solutions have been obtained in the form of a stream function from which the velocity fields and axial pressure gradient have been derived. The present analysis has been performed under long wavelength and low Reynolds number assumptions. The effects of various emerging parameters on the flow characteristics are shown and discussed with the help of graphs and the phenomenon of trapping is also discussed.

The contents of this chapter have been submitted for publication to: Journal of Biomechanics (2008).

Chapter I

Introduction

**On study of peristaltic motion of non-Newtonian fluids and
its biological applications**

VOLUME I

Introduction to Fluid Mechanics

by

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