Ain Shams University
University College for Women
Department of Mathematics

ON PROBLEMS OF CORDIALITY LABELING ON SOME GRAPHS

THESIS

Submitted in Partial
Fulfillment of Requirements for the Degree
of
MASTER OF SCIENCE (M.Sc.)
(PURE MATHEMATICS)

BY

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" و قل اعملوا فسيرى الله عملكم و رسوله و المؤمنون و ستردون إلى عالم

الغيب و الشهادة فينبئكم بما كنتم تعملون "

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M.Sc. Thesis

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إهداء

الحمد لله اللذى وفقنى لانجاز هذا العمل

وأتقدم بخالص الشكر لاساتذتى

و جميع أفراد أسرتى وبشكل خاص والدى "رحمه الله"

و أدعو الله أن يجعل هذا فى ميزان حسناته.

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SamahSaad

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ABSTRACT

Samah Saad Mahmoud Euat Allah.On problems of cordiality labeling on some graphs. Master of Science dissertation of Pure Mathematics (M.Sc.) , University College for Women , Ain Shams University.

The main purpose of this dissertation is to study the cordial labeling of some graphs. The dissertation consists of four chapters.

In the first chapter

We introduce some basic definitions and notations on the graph theory which are used in the following chapters. It includes the definitions of cordiality, second power graphs and the cartesian product of two graphs.

In the second chapter

We studied the cordiality labeling of the sum of two graphs one of them is a second power of a path and the other graph is a star graph which is not cordial in general. First, we started by reformulating the paper of Diab [5], and making some changes in his last result of this paper. Then, we generalize his paper for the sum of the second power of paths with other graphs namely $K_{1,m}$. We presented a necessary and sufficient conditions for the cordiality of $P_n^2 + K_{1,m}$. In fact, we proved that $P_n^2 + K_{1,m}$ is cordial if (n ,m) ≠ (4,odd) or (n ,m) ≠ (6 , odd) or (n ,m) ≠ (2, odd) or (n ,m) ≠ (3 ,1).

Through our proof we used our reformulating of Diab's previous result.

In chapter three

We studied the cordiality labeling of the union of two graphs the first one is a second power of a path and the other graph is a star graph. Also, we gave a necessary and sufficient conditions for the cordiality of $P_n^2 \cup K_{1,m}$. We showed that $P_n^2 \cup K_{1,m}$ is cordial for all n and m except (n ,m)≠ (2, odd) .

The results of this chapter and the previous chapter have been accepted for publication in "UTILITAS MATHEMATICA publishing INC " in Canada .

In chapter four

In this chapter , we studied the cordiality labeling of the Cartesian product of two graphs. The cordiality labeling of the product of two paths, a path and a cycle and a cycle and a path of special forms have been discussed. We gave a necessary and sufficient conditions for the cordiality of the Cartesian product graph $P_{2s+i}\times C_{2s+j}$. Also, we showed that the Cartesian product $P_{2s+i}\times P_{2s+j}$ and $C_{2s+i}\times P_{2s+j}$ for i, j =0,1 is cordial. We finished this chapter with some applications.

Key Words: Cordial labeling, Join cordial labeling, Union cordial labeling, Product cordial labeling.

SUMMARY

SUMMARY

The project in this thesis is based on the subject of graph theory, which studies graphs and operations on them.

The thesis consists of four chapters:

Chapter one :Basic concepts

It is an introduction and presents the main definitions of graph theory and basic concepts of the cordial labeling applied for some graphs which are used in our project.

Chapter two: The cordiality of the sum of the second power of paths and stars

In the first section of this chapter, we reformulate previous results of the cordiality of the join between paths and cycles. Also, we studied the cordial labeling of the sum of two graphs one of them is a second power of path and the other graph is a star graph which is cordial for all n, m except for (n ,m) = (4,odd) and (n ,m) = (6 , odd) and (n ,m) = (2, odd) and (n ,m) = (3 ,1).

The results of this chapter and the next chapter are accepted for publications in "UTILITAS MATHEMATICA publishing INC " in Canada .

Chapter three: Cordiality of the union of the second power of paths and stars

In chapter three, we studied the cordiality labeling of the union of two graphs the first one is a second power of path and the other graph is a star graph. We reached that P_n^2 U$K_{1,m}$ is cordial for all n and m except (n , m)≠ (2, odd) .

<u>Chapter four:</u> The cordiality of the product of two graphs

In this chapter , we studied the cordiality labeling of the Cartesian product of two graphs. The cordiality labeling of the product of some graphs has been studied and some theorems are obtained.

We showed in the first section that the Cartesian product graphs $P_{2s+i} \times C_{2s+j}$ are cordial for i, j =0, 1 . In the second section, we discussed the cordiality of the graphs $P_{2s+i} \times P_{2s+j}$. In the third section, The cordiality of $C_{2s+i} \times P_{2s+j}$ had been discussed. In the last section of this chapter, we showed some real applications based on graph labeling.

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