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التوثيق الالكتروني والميكروفيلم

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الأصلية تالفة

Eigenvalue distribution of integral operators defined by Sobolev-Orlicz kernels

Thesis
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Contents

Preface

It is our aim to demonstrate the great importance of the method of abstract operator theory to applications within the theory of integral operators.We apply the results on the eigenvalues of abstract operators on Banach spaces to determine the asymptotic distribution of the eigenvalues of integral operators in function spaces.

Why should one study integral operators ? The traditional answers are that integral equations have important applications outside of mathematics,and that they are the proper extension to analysis of the concepts and methods of the classical algebraic theory of the linear equations.A third possible answer is that the theory of integral operators is the source of all modern functional analysis and remain to this day a rich source of non-trivial examples.

The integral operators T usually map larger function spaces X (e.g. L_p - spaces) into smaller ones Y (e.g. Sobolev spaces W_p^s or Besov spaces B_{pq}^s).Usually the imbedding map I from Y into X belongs to one of the classes of Riesz operators.Since they have the ideal property,i.e. are stable under compositions with other operators.Hence the integral operator IT in X belongs to the same class and hence the eigenvalue results are applicable to IT.

In recent years,there has been emerged a new field of study in functional analysis: The theory of interpolation spaces.Interpolation theory has been applied to other branches of analysis (e.g. partial differential equations,numerical analysis , approximation theory).We want to use the real interpolation method to improve estimates of eigenvalues of integral operators defined by Sobolev-Orlicz kernels .

The asymptotic distribution of the eigenvalues of operators on function spaces defined by kernels usually depends on regularity properties and summability properties of the kernel, the smoother the kernel,the faster the eigenvalues tend to zero.It is shown in the books of König [K4] as well as Pietsch [P6] how to apply the theory of s-numbers and of p-summing operators together with interpolation of operators to give the precise asymptotics for the eigenvalues of kernels which belong to vector-valued Besov-spaces $B^{s_0}_{p,q}(B^{s_1}_{u,v})$, i.e. different smoothnesses in any of the two variables.

Since then,the real interpolation method has been generalized to interpolation theory with function parameter $(X_0, X_1)_{\rho,p}$ instead of $(X_0, X_1)_{\theta,p}$, replacing t^θ by some concave function ρ ,see Gustavsson-Peetre [GP] and Fernandez-Garcia [FG 2].The new method is particular well-suited to interpolation of Besov-Orlicz spaces $B^s_{\Phi,q}$.

The main aim of this thesis is to combine both methods to find optimal results for the asymptotic distribution of the eigenvalues of integral operators defined by kernels belonging to vector-valued of Besov-Orlicz spaces $B^{s_0}_{\Phi,q}(B^{s_1}_{\Psi,v})$ of the type that the eigenvalues belong to some Lorentz-Orlicz space $l_{\Phi,q}$ and find the best space of this type. The thesis consists of four chapters:

CHAPTER 1:

We collect the basic concepts and notations and some theorems which will be used in the subsequent chapters.

CHAPTER 2:

In this chapter we study the interpolation theory with function parameter.Next we study the interpolation of Lorentz spaces,Sobolev and Besov spaces,Schatten classes and (p,q)-summing operators.

CHAPTER 3:

Since our work is based on the distribution of the eigenvalues of integral operators on function spaces defined by kernels,we present the results on the eigenvalues of abstract operators on Banach spaces to determine the asymptotic distribution of the eigenvalues of integral operator T in function spaces Z.In particular we study kernels belonging to some vector-valued L_p-space(e.g. vector-valued Sobolev or Besov spaces).

CHAPTER 4:

We derive some lemmas and theorems for estimating the eigenvalues of integral operators between the Besov-Orlicz and the Sobolev-Orlicz spaces.

Finally,we give a good estimation for the eigenvalues of integral operators defined by kernels belonging to vector valued Besov-Orlicz spaces $B^{s_0}_{\Phi,q}(B^{s_1}_{\Psi,v})$.

Chapter 1

Introduction

Chapter 1

Introduction

In this introductory chapter we summarize some well known definitions and explain certain terminology used throughout this thesis.Also this chapter contains the definition and elementary properties of the Lorentz-sequence spaces $l_{p,q}$,the Besov spaces $B^s_{p,q}$, the Sobolev spaces W^s_p, the Orlicz spaces L_Φ and the (p,q)-summing operators $\Pi_{p,q}$. Further, the s-numbers of operators in Besov spaces is contained.

We use standard notations: $\mathbb{N}, \mathbb{Z}, \mathbb{R}$ and $\mathbb{C}$ for the natural, integer, real and complex numbers respectively.We let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.The Banach spaces will be abbreviated by X, Y, Z and sometime by A, B.By operators we mean continuous linear operators between Banach spaces. The space of continuous linear operators T from X to Y under the operator norm:

$$\|T\| \quad := \quad \sup\{\|Tx\|_Y \quad : \|x\| = 1\},$$

is denoted by $L(X, Y)$. The topological dual of X is denoted by X^* i.e.

$$X^* = L(X, \mathbb{K}) \qquad , \mathbb{K} \in \{\mathbb{R}, \mathbb{C}\},$$

and the duality pairing written $\langle x, x^* \rangle$ or $x^*(x), x \in X, x^* \in X^*$. The dual of the operator T is denoted by T^*. For $1 \leq p \leq \infty$ we denote by p^* the conjugate index defined by

$$\frac{1}{p} + \frac{1}{p^*} = 1 \qquad , 1 \leq p^* \leq \infty$$

Let Ω be a domain in the real Euclidean N - space $\mathbb{R}^N$ and X be a (real or complex) Banach space.Then $C(\Omega; X)$ is the collection of all vector- valued bounded and continuous functions on Ω, equipped with the norm

$$\|f\|_{C(\Omega;X)} \quad = \quad \sup_{x\in\Omega}\|f(x)\|_X. \tag{1.1}$$

Let $k \in \mathbb{N}$,then

$$C^k(\Omega;X) \quad = \quad \{f \in C(\Omega;X) : D^\alpha f \in C(\Omega;X), |\alpha| \le k\} \tag{1.2}$$

are Banach spaces equipped with the norm

$$\|f\|_{C^k(\Omega;X)} \quad = \quad \sum_{|\alpha|=k}\|D^\alpha f\|_{C(\Omega;X)} \tag{1.3}$$

where $\alpha = (\alpha_1, \ldots, \alpha_N)$ with $\alpha_j \in \mathbb{N}_0$ $, j = 1, 2, \ldots, N$ is a multi-index ,
$|\alpha| = \sum_{j=1}^N \alpha_j$ and

$$D^\alpha f(x) \quad = \quad \frac{\partial^\alpha f(x)}{\partial x_1^{\alpha_1}\cdots\partial x_N^{\alpha_N}} \qquad , x \in \Omega.$$

Furthermore , dx stands for the Lebesgue measure in $\mathbb{R}^N$,and

$$\|f\|_{L_p(\Omega;X)} \quad = \quad \left(\int_\Omega \|f(x)\|_X^p dx\right)^{1/p} \qquad , 0 < p < \infty,$$

with the usual modification

$$\|f\|_{L_\infty(\Omega;X)} \quad = \quad \sup_{x\in\Omega}\|f(x)\|_X.$$

If $X \in \{\mathbb{R}, \mathbb{C}\}$ we have the scalar- valued case.

1.1 Imbedding result for L_p -spaces

Theorem 1.1.1 *Suppose that vol* $(\Omega) = \int_\Omega dx < \infty$ *and* $1 \le p \le q \le \infty$. *Then* $L_q(\Omega) \subset L_p(\Omega)$ *with*

$$\|f\|_{L_p(\Omega)} \quad \le \quad (vol(\Omega))^{\frac{1}{p}-\frac{1}{q}}\|f\|_{L_q(\Omega)}.$$

Proof: Let $f \in L_q(\Omega), 1 \le q \le \infty$, then

$$\int_\Omega |f(x)|^p dx \quad \le \quad \left(\int_\Omega |f(x)|^q dx\right)^{p/q}\left(\int_\Omega dx\right)^{1-\frac{p}{q}}$$