



**Ain Shams University**

**Faculty of Engineering**

**Engineering Physics and Mathematics Department**

***Modeling of Passive Components on Silicon  
Substrates***

**A Thesis**

Submitted in Partial Fulfillment for the Requirements of the  
degree of Master of Science in Engineering physics.

Submitted By

**Mona Mohamed Amin Abdel Aziz El Sabbagh**

B.Sc. of Electrical Engineering  
(Electronics and Communications Engineering)  
Ain Shams University - 2000

Supervised By

**Prof. Dr. Omar Abdel Halim Omar**

**Dr. Mohamed Amin Dessouky**

Ain Shams University

**Cairo - 2005**



## **Examiners Committee**

Name: Mona Mohamed Amin El Sabbagh

Thesis: Modeling of Passive Components on Silicon Substrates.

Degree: Master of Science in Engineering physics.

Prof. Salah El Din Amin El Nahawi

Engineering Physics and Mathematics Department

Faculty of Engineering- Cairo University

Prof. Hani Fikry Ragaai

Electronics and Communication Engineering Department

Faculty of Engineering- Ain Shams University

Prof. Omar Abdelhalim Omar

Engineering Physics and Mathematics Department

Faculty of Engineering- Ain Shams University

Date:    /    / 2005

# C.V

Name of the researcher: Mona Mohamed Amin Abdel Aziz El Sabbagh

Date of Birth: 8<sup>th</sup> of July 1977

Place of Birth: Cairo

Degree: B.Sc. in Electrical Engineering

Department: Electronics and Communication Engineering

Faculty: Faculty of Engineering

University: Ain Shams University

Date of Degree: 2000

# Statement

This thesis is submitted to Ain Shams University in partial fulfillment of the degree of M.Sc. in Engineering Physics.

The work included in the thesis was carried out by the author in the Department of Engineering Physics and Mathematics, Ain Shams University.

No part of this thesis has been submitted for a degree or a qualification at any other University or institute.

Name: Mona Mohamed Amin Abdel Aziz El-Sabbagh

Signature:

Date:    /    / 2005

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# Abstract

For the design of integrated circuits in the radio frequency band, equally important to active elements are also passive ones. Besides capacitors and ohmic resistors, inductors were recently successfully integrated on chip. The existence of high quality inductors significantly determines the circuit performance.

Monolithic inductors are becoming of great importance for many RF circuit. They are used in low noise amplifiers for matching and as tuned band loads, in voltage controlled oscillators as part of their tank circuit and in power amplifiers. The integration of inductors on chip allows smaller chip size, lower power consumption, and low cost for integrated circuits.

The modeling of these inductors is very important for designers to allow them getting the best performance of their circuits. The present inductor performance is based on libraries, where the measured performance of prefabricated inductors is stored and can be used directly by the designers. Electromagnetic simulators are also used for inductor simulation, but are less frequently used due to their complexity and large simulation time. Some models have been introduced in the literature but most of them are based on fitting factors.

In this work, emphasis is on the study of different physical effects that dominate the performance of a planar inductor. These include the inductor shape; turn proximity effects, metal losses and electric and magnetic field losses in the substrate. The ultimate goal of this work is to build novel and efficient compact and scalable model for on-chip inductors on silicon that is based only on physical parameters to get the inductor performance. The model takes into account all loss mechanisms in metal, including skin effects, proximity effects and fringing effects. The model also includes the losses into the substrates due to electric and magnetic field penetration. The proposed model is compared with measurements and simulations for different processes and

geometrical parameters. Comparison shows that the model predicted exactly the maximum quality factor, the self resonance frequency, and the inductance variation near resonance. The model has no fitting parameters and the time of computation is only few seconds.

Based on the understanding of different physical effects that determine the inductor performance, and variation of this performance with frequency, an optimization algorithm is presented which allows the designer to get the needed inductance with best quality factor without having to investigate different interrelated physical parameters. The model and optimization algorithm are implemented in Matlab code.

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# List of Abbreviations:

CMOS	Complementary Metal oxide semiconductor.
BiCMOS	Bipolar- CMOS.
GaAs	Gallium arsenide
IC	Integrated circuit.
IF	Intermediate frequency
LNA	Low noise amplifier
PA	Power amplifier
PCB	Printed circuit board.
RF	Radio frequency.
RFICs	Radio frequency integrated circuits.
SRF	Self resonance frequency.
SGP	Solid ground plane.
TEM	Transverse electric and magnetic mode of propagation
VCO	Voltage controlled oscillator.