STABILITY OF BOUNDED HOLLOW JET PERVADED BY OBLIQUE MAGNETIC FIELD

A THESIS

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By

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SUMMARY

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Stability of Bounded Hollow Jet Pervaded by Oblique Magnetic Field

This thesis is mainly concerned with some stability problems of annular fluid jets acted upon by capillary, inertia, pressure gradient and electromagnetic forces.

Chapter I , is survey contains the concept of stability and the analysis in terms of normal modes technique. Also we have introduced the subject of magnetohydrodynamic (MHD) and explained the basic (MHD) equations of motion. We also did write about some previous studies relating to the work in this thesis.

Chapter II , is devoted to the instability of a streaming annular fluid jet surrounding a tar cylinder under general tenuous varying magnetic fields. The fluid jet is acted upon by the inertia , pressure gradient , capillary and electromagnetic forces . The equilibrium state of the model is studied and the fluid equilibrium pressure distribution is determined. The perturbation state has been analysed and the eigenvalue relation is derived. The latter is discussed and some limiting cases are recovered. It is found that the axial interior and exterior fields have strong stabilizing influences for axisymmetric m = 0 and non-axisymmetric $m \neq 0$ modes. The azimuthal varying field is purely destabilizing for m = 0 but in the $m \neq 0$ it is stabilizing or destabilizing according to restrictions. The streaming has a strong destabilizing influence in all modes for all wavelengths. Its influence increases the MFD unstable domains and decreases those of stability. As the tenuous azimuthal magnetic field influence is superior to those of axial fields, the MFD unstable domains are increasing with increasing q (the tar cylinder radius normalized

with respect to that of the fluid) values and vice versa. If the unperturbed fluid velocity is smaller than the Alfven wave velocity, the model destabilizing character is suppressed and stability arises.

In chapter III , we discussed the hydromagnetic stability of a gas jet penetrated a bounded liquid with non uniform magnetic field $\underline{H} = \left(0 \ , \frac{\beta H_0 r}{R_0} \ , \alpha H_0\right)$ while the magnetic field in the liquid region is being $\underline{H} = (0 \ ,0 \ , H_0)$ where α and β are some parameters and H_0 is the intensity of the magnetic field in the liquid region. The model is acted by the combined effects of the capillary, electromagnetic, inertia and pressure gradient forces. The problem is formulated, the perturbed and unperturbed states are studied, the boundary conditions are applied and finally the dispersion relation is derived and discussed analytically and numerically. Some reported works are recovered from the present result as limiting cases. The capillary force is completely stabilizing in the non-axisymmetric perturbation modes, while it is stabilizing or not in axisymmetric mode depending on whether the wavelength is longer or shorter than the circumference of the gas cylinder. The axial magnetic fields in the gas and liquid regions are strongly stabilizing. The azimuthal magnetic field is purely destabilizing in the axisymmetric mode, while in the non-axisymmetric perturbation modes it is stabilizing or not according to restrictions. Upon appropriate choices one may adapt these different effects so that the net stabilizing effects may be predominant over the destabilizing ones and stability sets in. Such results have many applications, in several domains concerning the natural gas and oil layers.

Some results of this study are published in the specializing journal Nuovo Cimento (Italy) Vol. 118B (2003) p. 713 – 723.

CHAPTER I

CHAPTER 1

INTRODUCTION

I.1. The concept of stability

The concept of stability is given recently by Chandrasekhar (1981). For the phenomenon which occurs in nature, it has to satisfy one more condition, namely that it must be stable to small disturbances. A smooth ball resting on the surface of a hemisphere is stable if the surface is concave upwards, but it is unstable to small displacements if the surface is convex upwards.

In fluid flows, smooth laminar flows are stable to small disturbances only when certain conditions are satisfied. Suppose, that we have a hydrodynamic system which, in accordance with the equations governing it, is in a stationary state, i.e. in a state in which none of the variables describing it is function of time. Let x_1 , x_2 , ……., x_k be a set of parameters which define the system. These parameters will include geometrical parameters such as the dimensions of the system; parameters characterizing the forces which may be acting on system, such as pressure gradient, temperature gradient, magnetic field, rotation, …..etc.

In considering the stability of such a system, we ask when the system is disturbed, will the disturbance gradually die down, or the disturbance will grows in amplitude. In the latter case we say the system progressively departs from the initial state and never reverts to it. In the first case, we say that the system could be reverted to the initial position with respect to a particular disturbance. In the second case it is said that the system is unstable, and this is the case even if there is only one special mode of disturbance with respect to which it is unstable. The system cannot be considered stable unless it is stable with respect to every possible disturbance to which it can be subjected.

If all initial states are classified as stable or unstable, according to the criteria state, there in the space of the parameters x_1 , x_2 , ……., x_k, the locus which separates the two classes of states defines the states of marginal stability of the system. By this definition, a marginal state is a state of neutral stability.

The locus of the marginal states in the $(x_1, x_2, \ldots\ldots, x_k)$ space will be defined by an equation of the form

$$\sum (x_1, x_2, \ldots\ldots, x_k) = 0 \qquad (1.1)$$

The determination of this locus is one of the prime objects of an investigation on hydrodynamic and magnetohydrodynamic stability.

In thinking of the stability of a hydrodynamic system it is often convenient to suppose that all parameters of the system, save one, kept constant while the chosen one is continuously varied. We shall then pass from stable to unstable states when the particular parameter, which we are varying, takes a certain critical value. We may say that instability arises at this value of the chosen parameter when all the others have their preassigned values.

States of marginal stability can be one of two kinds. The two kinds correspond to the two ways in which the amplitude of a small disturbance can grow or be damped : they can grow (or be damped) periodically; or they can grow (or be damped) by oscillations of increasing (or decreasing) amplitude. In the former case, the transition from stability to instability takes place via a marginal state exhibiting a stationary pattern of motions. In the latter case, the transition takes place via a marginal state exhibiting oscillatory motions with a certain definite characteristic frequency.

1.2. Magnetohydrodynamic basic equations

The basic equations which we need to describe essentially the interaction between hydrodynamics and electromagnetism, taken in a simplified (non-relativistic) version.

If a magnetized medium moves in the presence of magnetic field, an electric field is produced. If the medium is electrically conducting and different parts of it move at different velocities, the electric field will produce currents. This current interacts with the magnetic field and produces forces which, under the conditions discussed in the following, are strong enough to change the state of motion of the medium appreciably. In this way hydrodynamic motion and electromagnetic phenomena are coupled. We cannot use ordinary hydrodynamic equations or those of ordinary electrodynamic, but we must apply a combination which has been called magnetohydrodynamics (or simply magnetodynamics) equations.

To obtain such a set of equations (see Levich (1962), Woodson & Melcher (1968) and Lin (1976)) consider an element of fluid at position **r** and time **t** moving with velocity $\underline{u}(\underline{r}, t)$ and with mass density ρ. According to the conservation of mass, the equation of continuity is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0$$

or

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \underline{u}$$

(1.2)

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{u} \cdot \nabla$$

is the mobile operator (the operator following the motion).

The equation of motion which is the hydromagnetic Navier-Stokes equation: consider a viscous fluid moving with velocity $\underline{u}(\underline{r}, t)$ and subjected by viscosity , pressure gradient and electromagnetic forces. Then this equation could be written in the form

$$\rho\left(\frac{\partial}{\partial t}+\underline{u}\cdot\nabla\right)\underline{u}=-\nabla p+\underline{J}\times\underline{B}-curl\left(\nu\rho\ curl\ \ \underline{u}\right) \tag{1.3}$$

where ν is the kinematic viscosity coefficient, $\underline{J}$ is the current density and $\underline{B}$ is the magnetic induction vector.

If the electromagnetic variables are measured in SI units, Maxwell's equations are given by

$$\nabla\times\underline{E}+\frac{\partial\underline{B}}{\partial t}=0 \tag{1.4}$$

$$\nabla\cdot\underline{B}=0 \tag{1.5}$$

$$\nabla\times\underline{H}-\frac{\partial\underline{D}}{\partial t}=\underline{J} \tag{1.6}$$

$$\nabla\cdot\underline{D}=q \tag{1.7}$$

In our study we assume that there is no charge density (q = 0 i.e., the medium is free from resultant free charges). We also assume that the displacement current $\underline{D}$ to be zero which is true for slowly varying fields. Under these conditions, Maxwell's equations read