



Algorithmic Numerical Approach for Fitting Non-linear Statistical Models

Thesis

**Submitted in Partial Fulfillment for the
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Preparation in Science (Statistics)**

To

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Acknowledgments

Abstract

Abstract

The objective of this study thesis is to investigate the algorithmic approaches for fitting non-linear statistical models in its general form using numerical techniques, in addition to introducing some special forms of these models such as exponential, logistic, Poisson, and polynomial in addition to providing computer programs and flow charts for these programs.

Contents of thesis:

- (i) Presentation of the basic mathematical and statistical concepts, which are related to non-linear statistical models, application of non-linear statistical models, and introduction to neural networks, and its application in non-linear statistical models.
- (ii) Discussion of classical estimation methods for non- linear statistical models in univariate and multivariate cases, which are least squares method, the method of moments, maximum likelihood method, Bayesian estimation, and minimum chi-square method.
- (iii) Discussion of widely used numerical techniques in the estimation of the parameters of non-linear statistical models, with examples, and with computer programs and flow charts for these programs.

(v)

(iv) Discussion of the use of neural networks for fitting non-linear statistical models, with introductory computer program and its flow chart to explain how neural networks are used in fitting non-linear statistical models.

(v) Discussion of the application of neural networks in non-linear statistical models in particular the selection procedure.

Keywords: Estimation Methods, Numerical Methods, Non-Linear Regression Model, Neural Networks, Training algorithms, and Selection procedure.

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