

# **Methods of Accurate Measurement Of Passive Reactive Elements**

By

Eng. Heba Ahmed Mohamed Hamed

A THESIS

*Submitted in partial fulfillment of the  
requirements for the degree of M.Sc. in  
Electrical Engineering*

Supervised by:

Prof. Dr. Hamdy Salah Khalel El-Gohary  
Prof. Dr. Abla Hosni Abd El- Rahman  
Prof. Dr. Shendy Mohamed Ali El-Shal

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# **SUPERVISION SHEET**

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# STATEMENT

This thesis is submitted to Ain Shams University in partial fulfillment of the requirements for the degree of M.Sc. in Electric Engineering.

The included work in this thesis was carried out by the author at Department of the Electrical & Electronic Measurements, the National Institutes for Standards (NIS). No Part of this thesis has been submitted for a degree or a qualification at any other university or institution.

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To My Family  
To My Son  
To My Daughter

# ABSTRACT

**For accurately investigate the capability of the capacitance and inductance laboratory at NIS, this work describes a recent study on the performance of two types of accurate measuring methods, namely the classical bridge method and the electronic method. These methods are represented by three instruments that measure the standard capacitors and standard inductors. The first method is based on using both of the precision capacitance bridge assembly and the precision inductance bridge assembly. On the other hand, the second method makes use of the precision LCR meter. All these instruments are available in the capacitance and inductance laboratory at the National Institute for Standards (NIS), Egypt.**

**All of the bridge assemblies depend on one of the classical methods of measuring. The bridge method is based on comparison between the voltages and currents of the measured unit and a reference standard unit until a condition of balance is reached.**

**The precision LCR meter depends on electronic method of measurement which applies a default value of voltage across the measured unit. Then, it measures the voltage across the measured unit, the current flowing through it, and the phase angle between them. These three measurements are used to calculate several parameters which describe the measured unit.**

**Accurate measurements and calibrations of a group of standard capacitors and a group of standard inductors are carried out precisely through this study at two different frequencies (50 Hz and 1 kHz). All precautions have been performed which minimize the resulted error, such as the effect of temperature, humidity, electromagnetic interference ....etc. The uncertainty statement was reported for the measured values of capacitances and inductances according to the requirements of the international standard ISO/IEC 17025.**

**The measured values from both the bridges and the LCR meter show that the uncertainty of measured values of the instruments at**

**1 kHz frequency is much better than the accuracy of measured values of the instruments at 50 Hz by more than double.**

**The measured values at different frequencies and the investigated expanded uncertainties fairly demonstrate that bridges show high accuracy and less uncertainty of measured values at both frequencies. The precision LCR meter, although it is more recent and more sophisticated provides less accurate results. Even, the use of this meter saves time, effort and cost.**

**It was deduced that the NIS, Capacitance and Inductance laboratory finally has a level of uncertainty for inductance measurements better than 5 mH in range of Henry, 5  $\mu$ H at milli-Henry range, and 20 nH at range of micro-Henry for 1 kHz frequency and a level of uncertainty better than 10 mH in range of Henry, 10  $\mu$ H at milli-Henry range, and 25 nH at range of micro-Henry for 50 Hz frequency.**

**Also, It was deduced that the NIS, Capacitance and Inductance laboratory has a level of uncertainty for capacitance measurements better than 0.7 fF in range of pico-farad, and 1 nF in range of micro-Farad for 1 kHz frequency and a level of uncertainty better than 10 fF in range of pico-Farad, and 3 nF at range of micro-Farad for 50 Hz frequency.**

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