Ain Shams University

EFFICIENT NUMERICAL SOLUTION OF 3D INCOMPRESSIBLE VISCOUS NAVIER-STOKES EQUATIONS

THESIS
Submitted for the Degree
Of
DOCTOR OF PHILOSOPHY (Ph.D.)
(Pure Mathematics)

By

Belant Sabri Matit
Mathematics Department
University College for Women
Ain Shams University

Supervised
By

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Ain Shams University

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بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

وَمَن يَتَّقِ اللَّهَ يَجْعَل لَّهُ مَخْرَجاً (٢) وَيَرْزُقْهُ مِنْ حَيْثُ لَا يَحْتَسِبُ وَمَن يَتَوَكَّلْ عَلَى اللَّهِ فَهُوَ حَسْبُهُ إِنَّ اللَّهَ بَالِغُ أَمْرِهِ قَدْ جَعَلَ اللَّهُ لِكُلِّ شَيْءٍ قَدْراً (٣)

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عميد الكلية

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Dear Professor Matit,

I am happy to inform you that after a favourable referee report, your paper

„A numerical technique for the 3 - D Poisson equation"

has been accepted for publication in the International Journal of Pure and Applied Mathematics.

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CONTENTS

ABSTRACT

Berlant Sabri Matit. Efficient Numerical Solution of 3-D Incompressible Viscous Navier-Stokes Equations. Doctor of Philosophy (Ph.D.) of Pure Mathematics, University College for Women, Ain Shams University.

This thesis focuses on the finite difference methods for solving the incompressible Navier-Stokes equations. We deduce the vorticity-vector potential approach in three-dimensional space. A technique for the solution of the three-dimensional incompressible Navier-Stokes equations is presented, where the parabolic vorticity transport equation and the elliptic Poisson equation are discretized in a collocated Cartesian grid. Using the finite difference method, a direct and an iterative technique for the solution of the three-dimensional Poisson equation are derived. We compare the results of both techniques with the exact solution. With respect to the vorticity transport equation, both the explicit Euler and the implicit Crank-Nicolson methods are presented and compared. In addition, we study the different kinds of boundary condition as well as the stability of the numerical techniques. Finally, we present our time-marching

algorithm. This algorithm encapsulates the appropriate numerical techniques, boundary conditions and stability conditions. The correctness of the algorithm is demonstrated on the model problem of the lid-driven cavity in two- and three-dimensional space.

Keywords: Incompressible Navier-Stokes equations, discretization, finite difference, parabolic vorticity transport equation, elliptic Poisson equation, boundary conditions, stability conditions, time-marching algorithm, lid-driven cavity.

SUMMARY

Summary

This thesis focuses on presenting a finite difference technique for solving the incompressible Navier-Stokes equations in the three-dimensional space. The mathematical formulation adopted is of the vorticity-vector potential form.

Chapter 1 gives a brief introduction to the thesis topic. In chapter 2, we introduce the most common types of partial differential equations (elliptic, parabolic, and hyperbolic). Then, we briefly describe the numerical methods used for solving them. The basic concepts and techniques in the formulation of finite difference discretizations are presented. Examples are also included to illustrate important properties of the solutions of these equations.

In chapter 3, we derive the Navier-Stokes continuity and momentum equations. We explain how the continuity equation is deduced by applying the conservation of mass law to a fluid flow in Cartesian coordinates. The momentum equation is deduced by applying the conservation of momentum law to fluid flow.

We present, in chapter 4, a review of the previous literature related to our problem. We describe two important approaches: the primitive-variable approach and the vorticity-vector potential approach.

Chapter 5 describes a numerical method for solving incompressible N-S equations in 2-D Cartesian domain. Then, we generalize the described method for a 3-D Cartesian domain. In 2-D, equations for conservation of continuity and momentum are solved by using the vorticity-stream function approach. While this approach is appealing for 1- or 2-D problems, it gets more complicated for the 3-D case. The details of the 3-D vorticity-vector potential approach are derived in this chapter.

In chapter 6, we discuss four basic subjects related to the design of finite difference methods for 3-D steady incompressible viscous flows using vorticity formulations: boundary conditions, Poisson equation, vorticity transport equation and a time-marching algorithm. Some results of this chapter have been accepted for publication in the ***"International Journal of Pure and Applied Mathematics"***.

Chapter 7 gives the results of applying our numerical techniques on different model problems. First, the numerical and exact solutions of a 3-D Poisson equation are compared. Then, the complete time-marching algorithm for solving the incompressible N-S equations is applied on the model problem of the lid-driven cavity in 2- and 3-D.

Finally, concluding remarks and directions for future research are outlined in Chapter 8.

NOMENCLATURE