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for Art, Science and Education
Mathematics Department

NUMERICAL TREATMENT OF DIFFERENTIAL ALGEBRAIC EQUATIONS

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BY

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بِسْمِ ٱللَّهِ ٱلرَّحْمَـٰنِ ٱلرَّحِيمِ

قَالُوا۟ سُبْحَانَكَ لَا عِلْمَ لَنَآ إِلَّا مَا عَلَّمْتَنَآ ۖ إِنَّكَ أَنتَ ٱلْعَلِيمُ ٱلْحَكِيمُ (٣٢)

صَدَقَ اللَّهِ الْعَظِيم

Before I begin, I want to start with thanking my Lord "Allah" who is always teaching me and guiding me.

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Hybrid Special Class For Solving Differential-Algebraic Equations

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G-stable One -Leg Hybrid Methods for Solving ODEs and DAEs

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ABSTRACT

Abstract

Fatma Mohamed Yousry Mohamed. "Numerical Treatment of Differential Algebraic Equations." Doctor of Philosophy of Science dissertation (Pure Mathematics) .University College of Women for Art, Science and Education , Ain Shams University

The main purpose of this thesis is to study; the proposed numerical methods for solving ordinary differential equations and differential algebraic equations.

This thesis is divided into _six chapters_:

In _chapter 1,_ the definition of differential equation and it's sources are presented. Some fundamentals are mentioned such as, index, Index Reduction and Consistent initial values. Types of the differential algebraic equation are presented. Hessenberg forms are discussed.

In _chapter 2,_ numerical methods for solving ODEs and DAEs are discussed such as Runge-Kutta method, linear multistep method, Backward Differentiation Formulae, Extended BDF, Modified Extended BDF, Parametric class and its extended and Hybrid method. Order of MEBDF applied to DAEs.

In _chapter 3,_ three classes of hybrid methods to solve systems of differential algebraic equations (DAEs) and its stability analysis are introduced. These classes are based on a free parameter class of linear multistep method (LMM). Two classes contain one step point and one stage point (off-step point) of the first derivative of the solution. The third one contains two step points and one stage point of the first derivative of the solution.

In *chapter 4,* the one-leg twin of the first two hybrid classes in chapter 3 are studied for step k=2 and k=3. The order of convergence of these methods are determined according to the value of the parameters and compared to the order of convergence of their twin hybrid

multistep methods. The G-stability of these methods are studied. Finally, the methods are tested by solving DAEs.

Chapter 5 focuses on the implemented of the three hybrid classes and its twin one-leg methods on the implicit mixed differential algebraic equations. The orders of convergence for the above methods are discussed. Numerical tests are introduced.

In chapter 6 some practical problem are solved by the proposed classes which introduced in chapters 3 and 4.

Keywords: **Stiff ODEs; DAEs; Multistep Methods; BDF; Hybrid Methods; Stability Aspects ; One-leg Methods; G-Stability; Order of convergence.**

SUMMARY

SUMMARY

This thesis focuses on the numerical treatment of differential algebraic equations (DAEs). Many applications of circuit analysis, engineering and mathematical modeling lead to systems of DAEs; Some of these systems can be reduced to ordinary differential equation (ODEs) and then solved by ordinary differential equation methods. Other systems whose reduction to an explicit differential system is either impossible or impractical are solved directly. Throughout the thesis, some proposed methods for solving ODEs and DAEs are introduced; their stability and convergence are studied. Finally, some practical models are solved.

One of the main objectives for the research presented here is to investigate some of the numerical methods that have been developed for computing solutions to DAEs. We focus on Hybrid methods and their one leg methods for solving DAEs as this encompasses a wide variety of problems that appear in engineering studies.

The presented thesis consists of six chapters:

In ***chapter 1,*** the definition of differential algebraic-equation and it's sources are presented. Some fundamental concepts of DAEs are mentioned such as the classification of DAEs (Nonlinear DAEs , Linear DAE with constant coefficients, Linear time varying DAEs, Semi-explicit DAEs, Linear implicit DAEs, Fully-implicit DAE), the different definitions of index (differential index, perturbation index, tractability index, geometric index), index Reduction, consistent initial values and Hessenberg forms. At the end of this chapter, some applications are mentioned.

In *chapter 2*, numerical methods for solving ODEs and DAEs are mentioned such as Runge-Kutta method, linear multistep method, Backward Differentiation Formulae, Extended BDF, Modified Extended BDF, parameteric class of multistep method and its extended multistep method, 2+1 hybrid BDF, hybrid BDF methods (HBDF).

In *chapter 3*, three classes of hybrid methods to solve systems of differential algebraic equations (DAEs) are introduced. These classes are based on a free parameter class of linear multistep method. Two classes contain one step point and one stage point (off-step point) of the first derivative of the solution. The third one contains two step points and one stage point of the first derivative of the solution. The parameter is selected to improve the absolute stability regions. The proposed solution methodologies have larger stability regions compared to the backward differentiation formulae (BDF), the extended backward differentiation formulae (EBDF), and the Hybrid backward differentiation formulae (HBDF). The constructed first class is A-stable of the orders 2 to 5, and A(α)-stable of the orders 6 to 11. The second class is A-stable of the orders 2 to 5 and A(α)-stable of the orders 6 to 10. The last class is A-stable of the orders 3 and 4 and A(α)-stable of the orders 5 to 9. The A-stable methods of the three classes are L-stable. Numerical tests are conducted to validate the performance of the proposed technique.

In ***chapter 4***, an introduction to one-leg method and the definition of G-stability are mentioned. The One-Leg twin of the first two hybrid classes in chapter 3 are studied for steps k=2 and k=3. The order of convergence of these methods are determined according to the value of the parameters and compared to the order of convergence of their twin hybrid multistep methods. The G-stability of these methods is studied. The introduced methods were found to have some advantages over others numerical methods, such as backward differential formula (BDF).

In the first class, for $k = p = 2$, the one-leg twin has order 2 except when $s = (1/3)\ (-3 +\sqrt{3}\sqrt{(1 - 2\beta^* + \beta^*2)})$ it has order 3. For $k = p = 3$, the one-leg twin has order 2, however, if $\beta^*=0$, which leads to one leg hybrid BDF, or $s = 1$, which leads to the parameters class (4.41), it has order 3. In the second class, for $k = p = 2$, the one-leg twin has order 2 except when $s = -(-1+\beta^*)/\sqrt{3}$ it has order 3. For $k = p = 3$, the one-leg twin has order 2, however, if $\beta^* = 0$, which leads to hybrid BDF, or $s=0$, which leads to the parameters class (4.41) it has order 3. The corresponding one-leg twin of the two classes is G-stable for $k = 2$ and $k = 3$. The numerical tests show that the first class gives better results than the second.
The new results of this chapter are published in "Journal of Advances in Mathematics (JAM)", Vol. 5, No. 2 (2014) 711-722.

Chapter 5 focuses on the implementation of the three hybrid classes and its twin one-leg methods on the implicit mixed differential algebraic equations. The orders of convergence for the above methods are determined and some numerical tests are solved.

Finally, in Chapter 6, the new methods in chapter 3, 4 and 5 are used to solve models with differential-algebraic equations .The results are shown in figures and tables.