شبكة المعلومات الجامعية

بسم الله الرحمن الرحيم

شبكة المعلومـــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

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لم ترد بالأصـــل

Cairo University
Faculty of Science
Department of Mathematics

Properties of Certain Classes of Finite Groups

A thesis

submitted for the award of
the Ph. D. degree (Pure Mathematics)

By

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Abstract

A finite group G is said to be a T_0-group if every subnormal subgroup of $G/\Phi(G)$ is normal in $G/\Phi(G)$, where $\Phi(G)$ stands for the Frattini subgroup (the intersection of all maximal subgroups) of G. The principal aim of this thesis is twofold:

1- *To determine the structure of a finite group G all of its proper subgroups are T_0-groups.*

2- *To study the structure of a finite group G under the assumption that certain subgroups of prime power order are well-situated in G.*

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General introduction

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In this thesis only finite groups are considered. A group G is said to be a T_0-group if every subnormal subgroup of $G/\Phi(G)$ is normal in $G/\Phi(G)$, where $\Phi(G)$ stands for the Frattini subgroup (the intersection of all maximal subgroups) of G. The principal aim of this thesis is twofold:

1- *To determine the structure of a group G all of its proper subgroups are T_0-groups.*

2- *To study the structure of a group G under the assumption that certain subgroups of prime power order are well-situated in G.*

This thesis includes six Chapters:

CHAPTER I. This Chapter is concerned with establishing the notation and the basic definitions that will be used throughout the thesis.

CHAPTER II. We list a number of well known results which will be used throughout the thesis, referring the reader to their proofs in the literature. We also prove some of the easier ones that are used often in the thesis.

CHAPTER III. Following van der Waall and Fransman [**25**], we say that G is a T_0-group, if its Frattini quotient group $G/\Phi(G)$ is a T-group, where by a T-group we mean a group in which every subnormal subgroup is normal. It is clear that the class of T_0-groups contains the classes of T-groups and nilpotent groups. In [**16**], Gaschütz proved that every solvable T-group is a subgroup closed T-group (the group and all of its subgroups are T-groups). In contrast to Gaschütz's fact and the fact that every nilpotent group is a subgroup closed, we show by example that a solvable T_0-group is not a subgroup closed T_0-group (the group and all of its subgroups are T_0-groups).

The purpose of this Chapter is twofold:

(i) To present another characterization of solvable T-groups, and to give a sufficient condition for a group G to be a solvable T-group.

(ii) To determine the structure of a group G if it is not T_0-group but all its proper subgroups are T_0-groups.

CHAPTER IV. Following Kegel [**19**], we say that a subgroup of a group G is S-quasinormal in G if it permutes with every Sylow subgroup of G. In this Chapter we introduce a new subgroup embedding property called $\mathfrak{Z}$-permutable. It is closely related to S-quasinormality. We say that $\mathfrak{Z}$ is a complete set of Sylow