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شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

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On the representation problem of polyadic algebras and related structures

Presented by

Mohamed Khaled Mohamed Ismail Khalifa

A Thesis Submitted

to

Faculty of Science

In partial Fulfillment of the Requirements for

the Degree of M.Sc. of Science

(PURE MATHEMATICS)

Mathematics Department

Faculty of Science - Cairo University

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ABSTRACT

Student Name: Mohamed Khaled Mohamed Ismail Khalifa.

Title of the thesis: On the representation problem of polyadic algebras and related structures.

Degree: M. Sc. (Pure Mathematics)

In this thesis, we study the representation problem for finite dimensional polyadic algebras and related structures, including cylindric algebras and diagonal free algebras We will concentrate on several types of representations. One is the notion of complete representation, where a complete representation is one that preserves arbitrary meets (and joins) carrying them to set theoretic intersections (and unions). The other types of representations have to do with atom structures of algebras considered. These are the classes of strongly representable atom structures and weakly representable atom structures. We extensively use graphs and games as introduced in algebraic logic by Hirsch and Hodkinson.

keywords:

1- Quasi polyadic algebras.

2- Complete representation.

3- Strong representation.

Supervisors: **Signature**:

1- **Prof. Dr. F. W. Georgy**
Mathematics Department, Faculty of Science
Cairo University.

2- **Dr. T. Sayed Ahmed**
Mathematics Department, Faculty of Science
Cairo University.

Prof. Dr. M. Z. Abd-Alla
Chairman of Mathematics Department
Faculty of Science - Cairo University

27/3/2011

APROVAL SHEET FOR SUMISSION

Thesis Title: On the representation problem of polyadic algebras and related structures.

Name of candidate: Mohamed Khaled Mohamed Ismail Khalifa

This thesis has been approved for submission by the supervisors:

1- **Prof. Dr. F. W. Georgy**
Mathematics Department, Faculty of Science
Cairo University.

Signature: Georgy

2- **Dr. T. Sayed Ahmed**
Mathematics Department, Faculty of Science
Cairo University.

Signature: T. Sayed Ahmed

Prof. Dr. M. Z. Abd-Alla

Chairman of Mathematics Department

Faculty of Science, Cairo University

M. Z. Abd-Alla 27/3/2011

To the soul of my precious mother,

Table of Contents

List of Publications

1. T. Sayed Ahmed and Mohamed Khaled, Omitting types algebraically via cylindric algebras, International Journal of Algebra. 3(8) (2009) p.377-390.

2. T. Sayed Ahmed and Mohamed Khaled, On complete representations in algebras of logic, Logic Journal of IGPL. 17(3) (2009) p. 267-272.

3. T. Sayed Ahmed and Mohamed Khaled, Classes of algebras not closed under completions, Bulletin Section of Logic. 38 (1-2) (2009) p. 29-44.

4. T. Sayed Ahmed and Mohamed Khaled, Vaught's theorem holds for L_2 but fails for L_n when $n > 2$, Bulletin Section of Logic. 39(3-4) (2010) p. 15-22.

5. T. Sayed Ahmed and Mohamed Khaled, Omitting types and Complete representations, forthcoming in, Cylindric algebras part 3. Editors, Andréka, Férenszi and Németi. Springer. (2011).

In this thesis, the results in [2], and some of the results of [5] are included, and the results in [3] are further generalized. Some results of the thesis are not included in the above list, but are under preparation for submission.

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Introduction

In the middle of the nineteenth century George Boole initiated the investigation of a class of algebraic structures which were subsequently called boolean algebras. The theory of these algebras is directly related to the development of the most elementary part of mathematical logic. As is well known, however, the theory can be developed in a purely algebraic fashion, it has at present numerous connections with several branches of mathematics -set theory, topology and analysis- and hence it can be understood and appreciated by mathematicians unfamiliar with logical problems to which it owes its birth.

The work of Boole was the starting point for a continuous flow of inquiries into the algebraization of logic which led to the foundation of the theories of cylindric algebras, polyadic algebras and other related algebras. These theories, which are the main subject matter of our work, were originally designed to provide an apparatus for an algebraic study of first-order predicate logic, a portion of mathematical logic which plays a central role in contemporary logical research. Indeed, these theories do for predicate logic what has been done for sentential calculus by the theory of boolean algebras. In their subsequent development, however, these theories, just as that of boolean algebras, have founded interesting realizations and applications outside of logic. Also the theory can be presented in a manner primarily algebraic in character which will make the main bulk of this work fully accessible