A Unified Parametric Notion for Deterministic and Stochastic of Nonlinear Programming Problems

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Thesis
Submitted for the Degree of
Ph. D. of Science in Pure Mathematics

To
Mathematics Department
Faculty of Science
Cairo University
Giza, Egypt

2010

APPROVAL SHEET

Title of Ph. D. Thesis

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ACKNOWLEDGEMENTS

Acknowledgements

All the praise is to **ALLAH** whose guidance and the powers have given me forth to finish this work.

I am greatly indebted to thank Prof. Dr. **Mohamed S.A. Osman** for suggesting the problems in the present work, his fruitful guidance, valuable discussions, and absolute generosity in transferring his experience and endless support by day and night during driving this work.

I would like also to express his deepest thanks to Ass. Prof. Dr. **Mostafa M. El-Fahham**, for his fruitful suggestions, valuable directions, and precious support with deep experience

Also, I wish to express my deep gratitude to my supervisor Prof. Dr. **Assem Tharwat,** for his fruitful suggestions, and for his valuable discussions and support throughout the work on this thesis.

I would also like to thank all the staff members of the Mathematics Department, Faculty of Science, Cairo University, for all the help extended to me during this research work.

Finally, a special gratitude and appreciation to his family, without the continuous help and love of them this work would not have come to light.

Mohamed Ahmed Mosilhy

ABSTRACT

Abstract

Decision making is an integral part of our daily life. It ranges in scope from the individual to the largest groups and societies, including nations and, ultimately organizations at the global level. It considers situations ranging in complexity from the simple to the most complex involving multiple objectives. One of the criticisms of current decision making theory and practice is against the traditional approximation of multiple objective behaviors of men and organizations by a single technically convenient criterion.

In the last ten years, great concentrated effort has been done to develop methodology of sensitivity and stability analysis in nonlinear programming problem.

Sensitivity analysis in nonlinear programming includes the calculation of the rate of change of the solution vector and the objective value function with respect to the selected parameters due to small perturbations of these parameters near their initial values. Sensitivity analysis also includes the estimation of the parametric optimal value bounds for the problems of interest.

Stability analysis in nonlinear programming has been extensively investigated from the qualitative point of view.

One of the important studies of multiobjective optimization field is the stability of the problems. Different works had been done.

The first approach, presented important notions for stability of multiobjective optimization problems.

The second approach, provided a postoptimality analysis for problems that contain differentiable functions using the logarithmic penalty function.

The third approach, presented the studying of multiobjective optimization problems by analyzing the qualitative analysis of point-to-set maps that specify the efficient sets in both objective space and decision space using parameters in the feasible solution set and the domination structure separately.

In this thesis, developed theories, algorithms and new definitions are presented which can be applied to nonlinear programming problems involving multiple objectives. Such a multiplicity of objectives induces substitution of a single optimal solution by the whole set of efficient solutions.

INTRODUCTION

INTRODUCTION

One of the important studies of multi-objective optimization field is the stability and sensitivity analysis of the problems.

On the one hand, by stability we mean the studying of efficiency solution of a family of parameterized optimization problems in a wide range. On the other hand, by sensitivity (or differential stability) we mean studying of efficiency solution of a family of parameterized optimization problems in a small range.

A lot of papers talk about the stability and sensitivity, for example: Fiacco, A.V., and McCormick, G.P., (1968); Malanowski, K., Bank, Evans, Kornbluth, Hartely, Naacache, Jurkiewicz, Lucchetti, Pento, Yu, P.L., (1971); Tanino, T., Sawaragi, Y., Nakayama, H., Sakawa, M. and Yano, H., Osman, M.S., (1976); Guddat, Ammar, El-Banna, Youness, Saad, O., Tharwat, A., Radwan , Hannan, Gould, Gal, Wolf, Robinson, Wets, and Vogel, etc..

We will study three different approaches.

The first approach was started by, Osman, in (1976-1993) [53-62], which presented important notions for stability of multi-objective decision making problems by using a parametric approach. The existing results concerning the qualitative analysis of basic notions in convex programming problems with parameters in the objective function were reformulated to define and characterize the same notions for multi-objective convex

programming problems. Also, he presents an algorithm for decomposing the parametric space in multi-criteria convex programming problem using the nonnegative weighted sum approach. Furthermore, the notion of stability in convex programming problems with parameters in the objective function and the constraints are redefined and analyzed qualitatively for multi-objective convex programming problems.

The second approach was started by, Fiacco, in (1968-1990) [16-33], concerns essentially with the calculation of the parameter derivatives of the optimal value of the solution vector and the associated optimal Lagrange multipliers of general classes of parametric nonlinear programs satisfying standard ideal second order (twice continuous differentiability) assumptions. Non-algorithmic (problem oriented) and algorithmic (solution method oriented) results were given, the emphasis being ultimately on the provision of simple computable algorithmic methods and results that might be useful in practical applications. Also, he provided an effective approach for calculating parametric bounds on the optimal value function and a parametric feasible solution vector estimate, and also presented preliminary results that utilize the optimal value bounds to derive parametric solution vector bounds.

The third approach started by, Yu, in (1971-1976) [87,88,89], has introduced the concepts of domination structure

in the objective space and the non-dominated solutions with respect to it, and has provided a settlement for this problem.

Y. Sawaragi, H. Nakayama, and T. Tanino, in (1980, 1985) [81,82], present another approach for studying stability of multiobjective programming problems (vector optimization problems) by analyzing the continuity of the set-valued maps (point-to-set maps) that specify the efficient sets in both objective space and decision space by using parameters in the feasible solution set and the domination structure separately.

Also, T. Tanino, in (1988, 1990) [78, 79, 80], provides some results on the stability and sensitivity analysis in multiobjective nonlinear programming by the behavior of the perturbation map. The analysis was performed both qualitatively and quantitatively. The contingent derivatives of the perturbation map are also studied. Moreover, it is shown that the results can be refined in the convex case.

In this thesis, a developed theories and algorithms are presented which could be applied to nonlinear programming problems involving multiple objectives.

This thesis is organized in five chapters:

Chapter one, presents a quick view on some important definitions, theorems, and techniques for nonlinear programming, multiobjective nonlinear programming are presented that are important for the whole work.

Chapter two, provides the stability and sensitivity of Multiobjective Optimization Problem (MOP) by studying different approaches which we select three of them (first, second, and third approaches) and give a survey on them.

Chapter three, introduces a numerical example explain the stability using the three approaches. Also, the chapter provides the connection between the first and second approach by introduce a new algorithm which is a generalization of the algorithm in [1], and an illustrative example explain this algorithm. In addition, the chapter investigates the basic idea of the three approaches and a table of comparison of the approaches in terms of their applicability to the problem.

Chapter four, provides the new definition of P-stability notions of parametric multiobjective optimization problems and their properties. Also, the chapter presents the new definition of P-stability perturbation map and their properties. In addition, the chapter provides a new definition of stability perturbation map and their properties.

Finally, Chapter five, presents the conclusions and the recommendations for future work.