



Ain Shams University  
University College For Women  
Department Of Mathematics

# On some new graphs and their variation

THESIS  
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FULFILLMENT OF REQUIREMENTS  
FOR THE DEGREE  
OF  
MASTER OF SCIENCE  
(PURE MATHEMATICS)  
*BY*

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2-Difference Equations.	2h per week
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5-Theory of differential equations.	2h per week
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University College for Women  
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**M.SC THESIS  
(PURE MATHEMATICS)**

**TITLE OF THESIS:**

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جامعة عين شمس  
كلية البنات للأداب والعلوم والتربية  
قسم الرياضيات

# عن بعض أنواع المخططات الجديدة وتغيرها

رساله مقدمة من الطالبة

**نُجاة موسى حسين العبيدي**

قسم الرياضيات - كلية البنات  
جامعة عين شمس

**للحصول على درجة الماجستير**

في العلوم  
(رياضيات بحتة)  
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# SUMMARY

The project of this thesis concerns a field of mathematics called geometric topology.

The main purpose of this thesis is to study some new graphs and their variations.

The thesis is divided into five chapters:

## **Chapter One: Basic concepts**

In chapter (1) is an introduction and presents a brief survey of main important definitions that help us in our work.

## **Chapter two: The Graph Of Simplex Vertices**

In chapter (2) we will introduce a new types of graph. The representation of the new graph by adjacent and incidence matrices will be obtained. Some geometric transformations(Folding) on the new graphs are described.

**The results of this chapter were published in "Journal of Mathematics research" (Vol. 4, No. 1; February 2012) In Canada.**

## **Chapter three: The Retraction Of Graph with simplex Vertices.**

In chapter (3) we will discuss the retraction of new graph in which its vertices are simplexes. The limit of these retractions is obtained . Some theorems related to these transformations are proved. The effect of retraction on adjacent and incident matrices will be deduced.

**The results of this chapter are accepted for publication in "International Journal Of Computational And Applied Mathematics". In India.**

## **Chapter four: Neighborhoods Of The Undirected Graph With Simplex Vertices.**

In chapter (4) we will compute first and second neighborhood for new undirected graph with simplex vertices. Some theorems related to this subject are obtained. We also introduced some examples on 0-simplex,1-simplex,2-simplex and n-simplex.

**The results of this chapter are accepted for publication in "International Journal Of Applied Science and Technology". In United States Of America.**

## **Chapter five: Neighborhoods Of Directed Simplex Graph.**

In chapter (5) we will compute first and second neighborhood for directed simplex graph. Some theorems related to this subject are obtained. We also introduced some examples on 0-simplex,1-simplex,2-simplex and n-simplex.

**The results of this chapter were published in "International Journal Of Mathematics Archive "(Vol. 3, No. 4; April 2012) In India.**

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## **Arabic Summary**

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*Najat AL-Obaidi*



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

{ يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنْكُمْ  
وَالَّذِينَ أُوتُوا الْعِلْمَ كَرَجَاتٍ  
وَاللَّهُ بِمَا تَعْمَلُونَ خَبِيرٌ }

حَسْبُكَ اللَّهُ الْعَزِيزُ

# Chapter1

## Basic Concepts

In this chapter we will review briefly the basic topological and geometric concepts and results relevant to this work. We adapted these concepts to fit coherently into the framework of this thesis.

### 1.1 Graph:

There are many physical systems their performance depends not only on the characteristics of the components but also on the relative locations of the elements. An obvious example is an electrical network. If we change a resistor to a capacitor, generally some of the properties (such as an input impedance of the network) also change. This indicates that the performance of a system depends on the characteristics of the components. If, on the other hand, we change the location of one resistor, the input impedance again may change, which shows that the topology of the system is influencing the system's performance. There are systems constructed of only one kind of component so that the system's performance depends only on its topology. An example of such a system is a single-contact switching circuit. Similar situations can be seen in nonphysical systems such as structures of administration. Hence it is important to represent a system so that its topology can be visualized clearly.

One simple way of displaying a structure of a system is to draw a diagram consisting of points called "vertices" and line segments called "edges" which connect these vertices so that such vertices and edges indicate components and relationships between these components. Such a diagram is called a "Linear graph" whose name depends on the kind of physical system we deal with. This means that it may be called a network, a net, a circuit, a graph, a diagram, a structure, and so on.

Instead of indicating the physical structure of a system, we frequently indicate its mathematical model or its abstract model by a "Linear graph". Under such a circumstance, a linear graph is referred to as a flow graph, a signal flow graph, a flow chart, a state diagram, a simplicial complex, a sociogram, an organization diagram, and so forth. The earliest known paper on linear graph theory,

in 1736, is due to "Euler" who gave a solution to the "Konisberger" bridge problem by introducing the concept of linear graphs. In 1847, "Krichhoff" employed linear graph theory for an analysis of electrical networks, known today as the topological formulas for driving point impedance and transfer admittances. This probably is the first paper that applies the theory of linear graphs to engineering problems. However, it is not "Krichhoff's paper" but "Mobins conjecture", about 1840, concerning the four-color problem that seems to attract many scholars to devote themselves to linear graph theory.

In the past few years, graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging from operational research and linguistics to chemistry and genetics; at the same time it has also emerged as a worthwhile mathematical discipline in its own right[4]. The graph theory is being applied in many different fields such as engineering system science, social science and human relations, business administration and scientific management, political science, physical and organization systems, the electrical circuits and networks, route maps, architectural floor plans, chemistry, ecology, transportation theory, system diagnosis, music,.... [1].

## 1.2 An Abstract graph:

An "abstract" graph  $G$  is a diagram consisting of a finite non-empty set of elements, called "vertices" denoted by  $V(G)$  together with a set of unordered pairs of these elements, called "edges" denoted by  $E(G)$ . The set of vertices of the graph  $G$  is called "the vertex-set of  $G$ " and the list of the edges is called "the edge-list of  $G$ ". In other words, an "abstract" graph  $G$  is a pair  $(V(G); E(G))$  is a finite set and  $E(G)$  a set of unordered pairs of distinct elements of  $V(G)$ . Thus an element of  $E(G)$  is of the form  $(v_i, v_{i+1})$  where  $v_i$  and  $v_{i+1}$  belong to  $V(G)$  and  $v_i \neq v_{i+1}$ . The elements of  $V(G)$  are called "vertices" and the element  $(v_i, v_{i+1})$  of  $E(G)$  is called the "edges" joining  $v_i$  and  $v_{i+1}$  (or  $v_{i+1}$  and  $v_i$ ). The positions of the vertices and the length of the edges do not concern us; what is important is the number of vertices and the pairs of vertices which are connected by an edge[6]. Also the word "graph", here, refers to a diagram of points interconnected by lines, as shown in Fig (1.1), rather than to a picture representing a function[4].

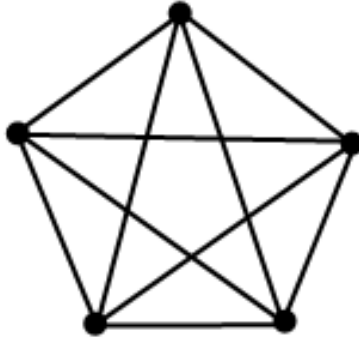
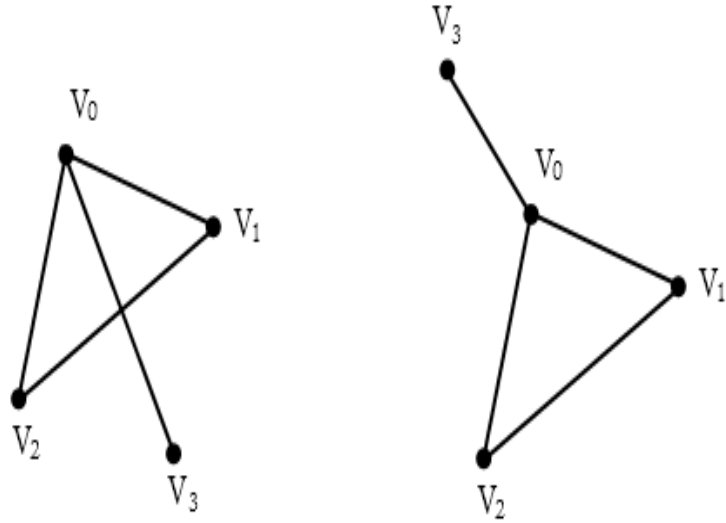


Fig (1.1)

The interconnections between points may refer to bonds between atoms in a chemical molecule, wires between terminals in an electrical network, roads between towns on a map, and so on [1,4].

### 1.2.1 Example:

Let  $G$  be a graph defined by  $V(G) = \{v_0, v_1, v_2, v_3\}$  and  $E(G) = \{v_0v_1, v_1v_2, v_2v_0, v_0v_3\}$ , then  $G$  may be represented by either Fig(1.2.a) or (1.2.b) where the later representation is more accurate[9].



Fig(1.2)

### 1.3 complete graph :

A graph in which every two distinct vertices are joined by one edge. The complete graph on  $n$  vertices is usually denoted by  $K_n$ , also  $K_n$  has exactly  $\frac{1}{2}n(n-1)$  edges. Fig.(1.3) shows the complete graphs  $K_n$  for  $n = 1, 2, 3, 4$ . The graph  $K_1$  is sometimes called the "trivial graph" [7,4].

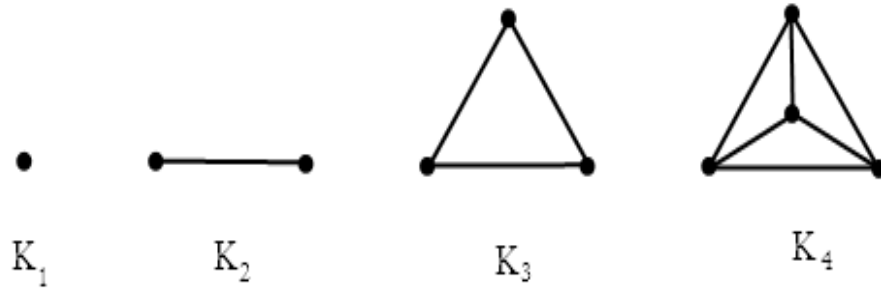


Fig. (1.3)

In other words, the "complete graph" is an abstract graph with the maximum number of edges. The complete abstract graphs with up to four vertices have realization in  $R^2$ , however it can be realized in  $R^3$ , as indeed can every abstract graph[1].

## 1.4 Null graph :

A graph which consists of a set of vertices and no edges is called a null graph[1].

*i.e.*, the set of edges in a null graph is empty, null graph is denoted on  $n$  vertices by  $N_n$ .

$N_4$  is shown in Fig.(1.4), note that each vertex of a null graph is isolated .



Fig.(1.4)

## 1.5 Path graph:

A path graph is a graph consists of a single path. The path graph with  $n$ -vertices is denoted by  $P_n$ , *Fig.(1.5)* [1].

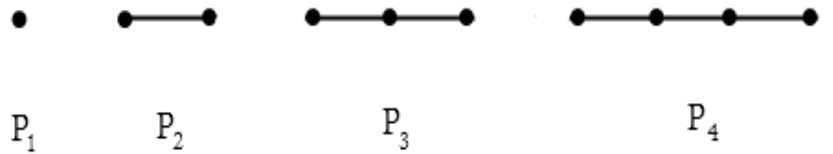


Fig.(1.5)