



Ain Shams University
Faculty of Engineering
Design and Prod. Eng. Dept.

STUDIES ON THE SENSITIVITY OF CONTROL CHARTS

By

Eng. / Mohamed Salah El-Din Akef

B. Sc. Mechanical Engineering
Military Technical College

*A Thesis Submitted in Partial Fulfillment for
the Requirements of the Degree of
Master of Science in Engineering*

Under the supervision of

Prof. Abd El-Latif Haridy
Professor
Design and Production
Engineering Department
Faculty of Engineering
Ain Shams University

Prof. Salah Zaki Abd El-Barr
Professor
Design and Production
Engineering Department
Faculty of Engineering
Ain Shams University

Dr. Tamer Adel Mohamed
Associate Professor
Mechanical Engineering Department
Faculty of Engineering
The British University in Egypt

2012

SUPERVISORS

Thesis Title:

STUDIES ON THE SENSITIVITY OF CONTROL CHARTS

Researcher Name:

Eng. Mohamed Salah El-Din Akef

Supervisors:

Name	Position	Signature
Prof. Abd El-Latif Haridy	Professor Faculty of Engineering Ain Shams University	
Prof. Salah Zaki Abd El-Barr	Professor Faculty of Engineering Ain Shams University	
Dr. Tamer Adel Mohamed	Associate Professor Faculty of Engineering The British University in Egypt	

Examiners Committee

A thesis submitted in partial fulfillment for the requirements of the degree of master
of science in engineering

Thesis Title: STUDIES ON THE SENSITIVITY OF CONTROL CHARTS

Researcher Name: Mohamed Salah El-Din Akef

Examiners :

Signature

Prof. Said Taha Mohamed Mouca
Professor Faculty of Engineering
EL-Mania University

Prof. Nahed Sobhi Abd El-Nour
Professor Faculty of Engineering
Ain Shams University

Dr. Tamer Adel Mohamed
Associate Professor
Faculty of Engineering
The British University in Egypt

/ / 2012

ACKNOWLEDGMENTS

First of all, great thanks to ALLAH. I would like to express my deepest sense of gratitude and thanks to my supervisors Prof. ABD EL-LATIF HARIDY and Prof. SALAH ELDIN ZAKI for their scientific assistance and guidance this research. Thanks are also extended to Dr. TAMER ADEL MOHAMED for his honesty in supervising this thesis, his great effort, his useful discussions and also for his scientific assistance throughout the whole work, he supported me all the time. My appreciation goes also to my friends for their encouragement and friendly attention. Finally, my thanks go to my parents for being there for me whenever I needed their support.

ABSTRACT

Very limited research available in literature evaluates the performance (rate of true and false alarm) of \bar{x} and R control charts with supplementary runs rules. This thesis studies the optimal combination of western electric supplementary runs rules in the design of control chart using computer simulation program to improve chart sensitivity. The main objective is to identify the best combination of runs rules that will reduce both in-control and out-of-control average run lengths, and thus, reduce chart errors, and achieve the average run length values pre-specified by the chart design when using \bar{x} -R control charts. This program is designed, developed and utilized for the joint design of \bar{x} and R control charts to control both process mean and variability, since the process may encounter changes or shift in the mean or variability simultaneously. This thesis, also, provides tables to be used by practitioners to specify the best combination of runs rules that will satisfy specific values for in-control Average Run Length (ARL_0) and out-of-control Average Run Length (ARL_1). In case that more than one combination satisfies the ARL_0 and ARL_1 specified by the designer, the simplest combination should be used. The computer program is designed under MATLAB 7.0.1 workspace

TABLE OF CONTENTS

TITLE THESIS	i
SUPERVISORS	ii
EXAMINERS COMMITTEE	iii
ACKNOWLEDGMENTS	iv
ABSTRACT	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	viii
LIST OF TABLES	x
LIST OF SYMBOLS	xi
CHAPTER ONE : INTRODUCTION	1
CHAPTER TWO : LITERATURE REVIEW.....	4
2.1 Economical Design Of Control Charts.....	6
2.2 Economical - Statistical Design Of Control Charts.....	7
2.3 Statistical Design Of Control Chart.....	9
CHAPTER THREE: METHODOLOGY.....	12
3.1 Computer Program Flow chart.....	12
3.2 Computer Program.....	15
3.2.1 Input Data.....	16
3.2.2 Selection Code.....	17
3.2.3 Computation of The Process Parameters.....	18
3.2.4 Generation of Random Number and Sampling.....	20
3.2.5 Analysis of zones Rules for Control Charts.....	21
3.2.5.1 Zone 1 Rule 1.....	22
3.2.5.2 Zone 2 Rule 2.....	23
3.2.5.3 Zone 3 Rule 3.....	24
3.2.5.4 Zone 4 Rule 4.....	26
3.2.6 Computation of The Average Run Length for Control Charts.....	29
3.2.7 Program End.....	29
3.3 Number of Random Trials.....	30
3.3.1 Simulation Model Validation.....	31
CHAPTER FOUR: RESULTS AND DISCUSSION.....	32
4.1 Fixing Standard Deviation Ratio and Varying Standardized Shift Values	32
4.1.1 Standard deviation ratio =1.....	33
4.1.2 Standard deviation ratio = 0.9.....	37
4.1.3 Standard deviation ratio = 0.8.....	41
4.1.4 Standard deviation ratio = 0.75.....	44

4.1.5	Standard deviation ratio = 0.5	47
4.2	Fixing Standardized Shift Values and Varying Standard Deviation Ratio	50
4.2.1	Standardized shift values = 0	50
4.2.2	Standardized shift values = 0.3	54
4.2.3	Standardized shift values = 0.5	57
4.2.4	Standardized shift values = 1.0	60
4.2.5	Standardized shift values = 1.5	63
4.2.6	Standardized shift values = 2.0	66
4.2.7	Standardized shift values = 3.0	69
4.3	Practitioners Table	72
4.3.1	Numerical Example	74

CHAPTER FIVE: SUMMARY, CONCLUSIONS AND FUTURE RESEARCH...75

REFERENCES.....78

APPENDIX..... 81

LIST OF FIGURES

Figure (2.1): Atypical control chart.....	4
Figure (3.1): Program flow chart.....	12
Figure (3.2): “the process parameters mean and standard deviation”.....	17
Figure (3.3): “the number of random trials”.....	17
Figure (3.4): “a sample size of n. As well as, control factors according to the sample size “.....	17
Figure (3.5): “standardized shift value (δ) and standard deviation ratio (λ)”.....	17
Figure (3.6): “selecting the rule code”.....	18
Figure (3.7): “computation of the process parameters”... ..	19
Figure (3.8): “computation of UCL and LCL”... ..	19
Figure (3.9): “computation of two-sigma limits”.....	19
Figure (3.10): “computation of one-sigma limits”.....	20
Figure (3.11): “computation limits for the R-chart”.....	20
Figure (3.12): “random sample generation”.....	21
Figure (3.13): “rule 1 \bar{x} -chart”.....	22
Figure (3.14): “rule 1 R-chart”.....	23
Figure (3.15): “rule 2 \bar{x} -chart”.....	23
Figure (3.16): “rule 2 R-chart”.....	24
Figure (3.17): “rule 3 \bar{x} -chart”.....	25
Figure (3.18): “rule 3 R-chart”.....	26
Figure (3.19): “rule 4 \bar{x} -chart”.....	27
Figure (3.20): “rule 4 R-chart”.....	28
Figure (3.21): “computation of the average run length”	29
Figure (3.22): “program end”.....	29
Figure (3.23): Effect of changing of replication on the ARL.....	30
Figure (4.1): Standard deviation ratio = 1.0.....	36
Figure (4.2): Standard deviation ratio = 0.9.....	40
Figure (4.3): Standard deviation ratio = 0.8.....	43
Figure (4.4): Standard deviation ratio = 0.75.....	46

Figure (4.5): Standard deviation ratio = 0.5.....	49
Figure (4.6): Standardized shift values = 0.....	53
Figure (4.7): Standardized shift values = 0.3.....	56
Figure (4.8): Standardized shift values = 0.5.....	59
Figure (4.9): Standardized shift values = 1.....	62
Figure (4.10): Standardized shift values = 1.5.....	65
Figure (4.11): Standardized shift values = 2.....	68
Figure (4.12): Standardized shift values = 3.....	71

LIST OF TABLES

Table (3.1): Comparison with ANDREW C. PALM (1990).....	31
Table (4.1): The results of ARL for runs rules at standard deviation ratio $\lambda = 1$	34
Table (4.2): The results of ARL for runs rules at standard deviation ratio $\lambda = 0.9$	38
Table (4.3): The results of ARL for runs rules at standard deviation ratio $\lambda = 0.8$	42
Table (4.4): The results of ARL for runs rules at standard deviation ratio $\lambda = 0.75$	44
Table (4.5): The results of ARL for runs rules at standard deviation ratio $\lambda = 0.5$	47
Table (4.6): The results of ARL for runs rules at standardized shift values $\delta = 0$	52
Table (4.7): The results of ARL for runs rules at standardized shift values $\delta = 0.3$	55
Table (4.8): The results of ARL for runs rules at standardized shift values $\delta = 0.5$	57
Table (4.9): The results of ARL for runs rules at standardized shift values $\delta = 1.0$	60
Table (4.10): The results of ARL for runs rules at standardized shift values $\delta = 1.5$	63
Table (4.11): The results of ARL for runs rules at standardized shift values $\delta = 2.0$	66
Table (4.12): The results of ARL for runs rules at standardized shift values $\delta = 3.0$	69
Table (4.13): The results of ARL for runs rules at both different standardized shift values and standard deviation ratio	72
Table (4.14): Results of Numerical Example.....	74

LIST OF SYMBOLS

ARL	Average Run Length.
ARL_0	In-control Average Run Length.
ARL_1	Out-of-control Average Run Length.
ATS	Average time to signal
h	The time interval between samples.
K	The width coefficient of the control limits.
LCL	Lower control limit.
n	The sample size.
UCL	Upper control limit.
SPC	Statistical process control.
VSI	Variable sampling interval
λ	Standard deviation ratio.
δ	Standardized Shift Values
α	Type I error.
β	Type II error.

CHAPTER ONE

INTRODUCTION

Statistical process control (SPC) is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability. These tools are often called "the magnificent seven". Among these tools, come the control charts which were developed in the 1920s by Shewhart (1939) to monitor process. The most famous and common Shewhart chart is the \bar{x} -chart which is used to monitor the process mean due to its simplicity for understanding and operating. Montgomery (2001) explained that control charts may serve, first, to define the goal or standard for a process that management strive to attain; second, they may be used as an instrument for attaining that goal; and third, they may serve as a means of judging whether the goal has been reached. The use of control charts results in two different types of errors; namely, type I error denoted as (α) , and type II error denoted as (β) . Type I error is the conditional probability that a point falls outside the control limits given the process is in-control, also known as probability of false alarm. Type II error is the conditional probability that a point falls inside the control limits given the process is out-of-control. The power, or effectiveness, of a control chart is usually measured by $(1-\beta)$, which is the conditional probability that a point falls outside the control limits given the process is out-of-control. The Average Run Length (ARL) is the average number of inspected samples required to signal an out-of-control. When a process is out-of-control, the users want the control chart to signal quickly, i.e. to have a small out-of-control ARL, called ARL_1 . Conversely, when the process is in control, the users want the chart to produce fewer false alarms, i.e. to have a large in-control ARL, called ARL_0 .

For any control chart, the ARL can be calculated as follows:

$$ARL = \frac{1}{p(\text{out of control signal})} \quad \text{Montgomery (2001)}$$

Thus, when a process is out-of-control, the out-of-control ARL can be calculated as follows:

$$ARL_1 = \frac{1}{1 - \beta} \quad \text{Montgomery (2001)}$$

Conversely, when the process is in-control, the ARL be calculated as follows:

$$ARL_0 = \frac{1}{\alpha} \quad \text{Montgomery (2001)}$$

A control chart may indicate an out-of-control condition either when one or more point falls beyond the control limits, or when plotted points exhibit some nonrandom pattern of behavior. The Western Electric Handbook (1956) suggests a set of decision runs rules for detecting nonrandom patterns on control chart. The use of each rule or combination of them improves the sensitivity of the control chart to detect process shifts. Specifically, it suggests concluding that the process is out-of-control if one of the following rules occurs :

1. One or more points outside of the control limits.
2. Two of three consecutive points outside the warning limits, but still inside the control limits.
3. Four of five consecutive points beyond from the center line.
4. A run of eight consecutive points on one side of the center line.

These criteria are often used in practice for enhancing the sensitivity of control charts to a small process shift, so that the chart may respond more quickly to the assignable causes. However, increasing the chart response to out-of-control signal also increases the probability of false alarm, i.e. reducing β will result in increasing α and vice versa. The main objective of this research is to identify the best combination of runs rules that will reduce both in-control and out-of-control average run lengths. This thesis develops computer program to facilitate achieving objective and improve chart sensitivity, a computer program is designed and developed under MATLAB 7.0.1 workspace to compute the ARL_0 and ARL_1 where different combinations of runs rules (mentioned previously from one through four) are utilized. The program is designed to be used for joint \bar{x} and R control charts (control chart for mean and control chart for range). However, it can be used for either one separately. A comprehensive review of the literature pertaining for three design approaches were performed, these are economic, economic-statistical and statistical approaches. This is shown in Chapter 2 of the thesis following the introduction. Chapter three presents the computer program. Chapter four represents tables to be used by practitioners to specify the best combination of runs rules. Chapter five presents summary, conclusion and future research.

CHAPTER TWO

LITERATURE REVIEW

Control charts are the simplest type of statistical process control procedure. They are often used to monitor a parameter (or parameters) of the distribution of a quality characteristic of items as they are produced. Also, considering control chart as a monitoring tool for the process mean, recently arose in the context of supplier and customer relations. The supplier is keeping control charts on important product properties and successfully using them to make process improvements Cham and Woodall (1987). A typical control chart is shown in Fig.(2.1).

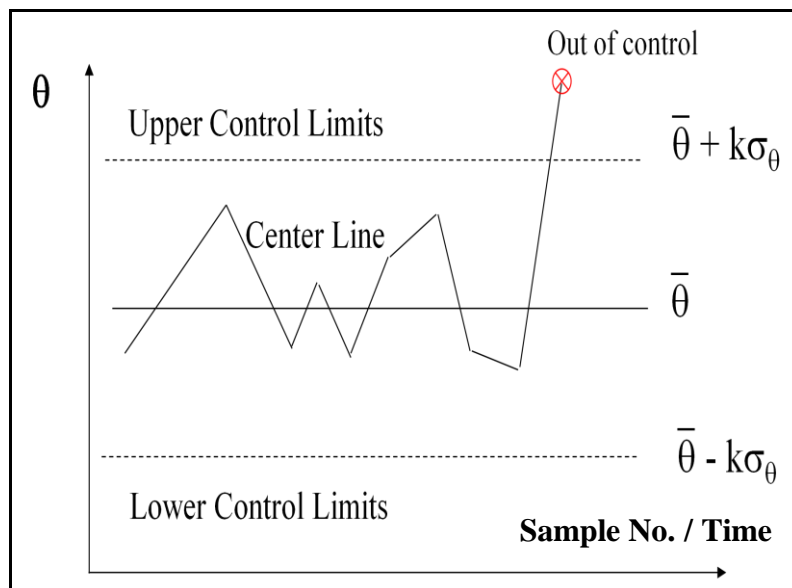


Figure (2.1) a typical control chart