The Elements of Thesis

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Approval Sheet

Symmetric Analysis for some Partial Differential Equation

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SUMMARY

Chapter I

This chapter is an introduction to the group analysis. It includes the main concepts of the invariant solutions and the theorems and definitions needed for the study of the differential equations' invariant properties. It also introduces the contact transformations for evolution-type equations and the main theorems needed for the application of the method.

In addition, it presents the optimal system of subalgebra and the introduction of fractional differential equations which contain some definitions and properties of Riemann-Liouvill, Capito and their aspects.

Chapter II

This chapter contains new classes of exact solution for a generalized Zakharov equations (GZE) which are considered as a set of coupled equations, a generalized Ito system of four coupled nonlinear evolution equations and (2+1)-dimensional higher order Broer-Kaup system.

In the first class of methods, we found that both the Lie groups' analysis and the contact transformations lead to the same Lie point transformation generators. These methods reduce the above three systems to ordinary differential equations which are solved analytically.

Chapter III

In this chapter, we aim to determine the optimal subalgebras and correspond reductions of the generalized Zakharov equations (GZE), a generalized Ito system of four coupled nonlinear evolution equations and (2+1)-dimensional higher order Broer-Kaup system to ordinary differential equations by using Ibragimov's variant schemes.

By solving these ordinary differential equations, we will obtain new classes of the problems' solution.

The contents of Section (3.5) is published in (International conference on

mathematics, trends and development ICMTD12, Cairo, Egypt, 27-29 Dec.(2012) [40])

Chapter IV

In this chapter, we investigate the Q-symmetry for Boussinesq equation which belongs to the KdV family of equations and describes the motions of long waves in shallow water under gravity propagating in both directions, the classical Drinfel'd–Sokolov–Wilson equations (DSW) and a generalized Ito system of four coupled nonlinear evolution equation to obtain the Lie point transformation generators. It is obtained from symmetry and contact transformations which are stated in chapter II. The application of one-parameter group reduces the number of independent variables and consequently the problems to set of ordinary differential equations (ODEs) which are solved analytically.

Also, we investigate the conditional Q-symmetries for Boussinesq equation, Drinfel'd–Sokolov–Wilson (DSW) system and a generalized Ito system. Reductions to ordinary differential equations are performed by using these symmetry properties.

The contents of Section (4.2.5) and (4.3.5) is published in (International Journal of differential equations, (2013) In Press.[41])

The contents of Section (4.2.6) and (4.3.6) is published in (International conference on mathematics, trends and development ICMTD12, Cairo, Egypt, 27-29 Dec.(2012) [39])

Chapter V

In this chapter, We deal with fractional analysis through studying some relating definitions and properties such as; the definitions of Riemann-Liouvill, Capito and their aspects. It also includes a large number of examples in order to construct symmetries of FDE and use these symmetries in constructing the exact solutions of equations.

The list of systems in thesis

The generalized Zakharov equations (GZE) which are a set of coupled equations:

$$i\frac{\partial E}{\partial t}+\frac{\partial^2 E}{\partial x^2}-2\beta\left|E\right|^2 E+2nE=0,$$
$$\frac{\partial^2 n}{\partial t^2}-\frac{\partial^2 n}{\partial x^2}+\frac{\partial^2\left|E\right|^2}{\partial x^2}=0.$$

A generalized Ito system of four coupled nonlinear evolution equations which was introduced by:

$$u_t=v_x,$$
$$v_t=(-2v_{xxx}-6(uv)_x+(bp+f)w_x+(cw+g)p_x),$$
$$w_t=(w_{xxx}+3uw_x),$$
$$p_t=(p_{xxx}+3up_x),$$

where b, c, f and g are arbitrary constants.

(2+1)-dimensional higher order Broer-Kaup system

$$u_t+4\left(u_{xx}+u^3-3\,\mathrm{u}u_x+3uw+3p\right)_x=0,$$
$$v_t+4\left(v_{xx}+3vu^2+uv_x+3vw\right)_x=0,$$
$$w_y-v_x=0,$$
$$p_y-(uv)_x=0.$$

Boussinesq equation which belongs to the KdV family of equations and describes motions of long waves in shallow water under gravity propagating in both directions, is given by

$$u_{tt}+u_x^2+uu_{xx}+u_{xxx}=0.$$

The classical Drinfel'd–Sokolov–Wilson equations (DSW)

$$u_t+pvv_x=0,$$
$$v_t+qu_{xxx}+ruv_x+svu_x=0,$$

where p,q,r,s are some nonzero parameters.

CHAPTER I

INTRODUCTION TO LIE GROUP AND FRACTIONAL DIFFERENTIAL EQUATIONS

1.1 Introduction

The Mathematical models of many problems in mechanics and engineering science are often described by differential models either linear or nonlinear partial differential equations. It is well known that the problem of group classification of a given family of equations (containing arbitrary parameter or functions) is more complicated than the problem of calculation of a symmetry group for a given equation and gives a systematic approach to obtain analytic solutions for partial differential equation system [62].

In the study of partial differential equations, the discovery of explicit solutions has great theoretical and practical importance. In the case of linear system, general solutions can be built up by superposition from separable solutions; for nonlinear systems, explicit solutions are used as models for physical or numerical experiments, and often reflect the asymptotic behavior of more complicated solutions. Over ten years, a variety of methods for finding these special solutions by reducing the partial differential equation to one or more ordinary differential equations have been devised. Included are the method of group invariant solutions.

Group-theoretic method are powerful, versatile and fundamental to the development of systematic procedures that lead to invaginate solution of differential equations since they are not based on linear operators superposition. We applied a one-parameter Lie group method not only to find invariants of partial differential equations but also to predict the existence of invariants to obtain invariants and partially invariant solution to form the group in order to transform a given partial differential equation to less complicated or ordinary differential equation [34], [66] and [71].

A systematic investigation of continuous transformation groups was carried out by Lie (1882-1899). His original goal was the creation of a theory of integration for ordinary differential equations analogous to the Abelian theory for the solution of algebraic equations. He investigates the fundamental concept of the invariance group admitted by a given system of differential equations. Today, the mathematical approach whose object is the construction and analysis of full invariance group admitted by a system of differential equations is called group analysis of differential equations. These groups, now usually called Lie groups and the associated Lie algebra's have important real world applications.

Most of the required theory and description of the techniques of this method can be found in references [31], [33], [34] and [71]. The text of Bluman and Cole is a good exposition of the application to both ODE's and PDE's. The books by Bluman and Kumei and Setphani describe, as does Oliver's. The problem of classifying the subgroups and reducing to optimal systems takes on more importance for multidimensional PDE's. The group classification problem is that of determining the groups allowed by differential equations containing an arbitrary parameter or function. That is, finding the particular group associated with particular values or forms of these arbitrary parameters or functions.

Unfortunately, for systems of partial differential equations, the symmetry group is usually of no help in determining the general solution (although in special cases it may indicates when the system can be transformed into a more easily solvable system such as a linear system). However, one can use general symmetry groups to explicitly determine special types of solutions which are themselves invariant under some subgroup of the full symmetry group of the system. These group-invariant solutions are found by solving a reduced system of differential equations involving fewer independent variables than the original system.

Nonlinear phenomena appear in many areas of scientific fields such as solid state physics, plasma physics, fluid dynamics, Mathematical biology, engineer, optical fibers, geochemistry and chemical kinetics. The nonlinear problems are characterized by dispersive effects, dissipative effects, convection-advection, diffusion process and other effects.

Because of the increased interest in the theory of solitary waves and nonlinear phenomena, there has made noticeable progress in the construction of the exact solutions of