STABILITY OF MAGNETIZED SUPERPOSED AND COMPRESSIBLE FLUIDS

Thesis Submitted in Partial Fulfillment
for Ph.D. Degree in Engineering Mathematics

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2008

ABSTRACT

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Title : " STABILITY OF MAGNETIZED SUPERPOSED AND COMPRESSIBLE FLUIDS"

Degree : Doctorate of Philosophy in Engineering Mathematics.

Faculty : Faculty of Engineering, Ain Shams University.

Specialty : Engineering Physics and Mathematics (Engineering Mathematics).

In the last decades, the MHD stability of fluids problems have received great attention for their crucial applications in various domains of science. Here we study the stability of different geometrical models analytically and numerically taking into account the effect of different external forces such as electromagnetic, surface tension, selfgravitational,… etc.

In Chapter (I), we did write down about the concept of stability with a brief discussion of the techniques which may be used in stability problems analysis focusing on the normal mode method. Also we did write down the basic hydrodynamics and magnetohydrodynamics (MHD) equations which are essential for formulating all stability problems. Different boundary conditions at the fluid interfaces are

explained. Finally, we review on some reported works in the hydrodynamics and magnetohydrodynarmics stability of jets and other models. Survey concerning some discussion about superposed fluids have been carried out.

In Chapter (II), The hydromagnetic instability of compressible hollow jet involved with surface tension is discussed in the axisymmetric mode for all short and long wavelengths. The dispersion relation is derived and discussed analytically and numerically. The axial magnetic fields inside the gas and liquid regions have stabilizing effects for all short and long wavelengths. This is physically interpreted that the axial field exerts a strong effect which causes the bending and twisting of the magnetic lines of force. The compressibility effects need careful treating. Here the incompressible fluid result is obtained as a tends to ∞ (a is the sound speed in the fluid). For finite value of a (i.e. compressible fluid), the temporal amplification is larger than that in the incompressible case. So the compressibility has a strong destabilizing tendency and increase the unstable domains. The streaming is destabilizing for all short and long wavelengths. The capillary force is destabilizing for small wave numbers while it is stabilizing for all the rest wavelengths. Whatever the stabilizing effect of the electromagnetic force is strong enough, the capillary, streaming and compressible instability effects could not be suppressed and the model will be always unstable.

The results of this problem have been published in 3rd International Conference on Engineering Mathematics and Physics, Military Technical College, Cairo, Egypt, May 16-18, (2006) pp.47-66.

In Chapter (III), The self gravitating instability of fluid cylinder penetrated by toroidal varying magnetic field internally has been developed. Upon using the linear perturbation technique, the problem is studied, the dispersion relation is established and discussed. Some reported works are recovered from the present general data as limiting cases with suitable simplifications.. The electromagnetic force has stabilizing effect for all perturbed wavelengths. The uniform magnetic field penetrated in the tenuous medium has no direct influence on the stability of the model. The self gravitating force is stabilizing for very short wavelengths but it is destabilizing otherwise . The magnetic field influence decreases the self gravitating destabilizing character but never suppressed it. This is due to the fact that the gravitational instability of sufficiently long waves will persist and the reason for that lies in the logarithmic singularity of the gravitational potential energy for infinite wavelengths.

The results of this problem have been published in Journal of the Faculty of Education, Ain Shams University, Cairo, Egypt,31 (2006) 427-438.

In chapter (IV), we extend our previous recent work (Radwan and Hussain (2006)) to investigate the non-linear stability of a liquid cylinder acting upon the combined effect of the inertia, capillary and electromagnetic forces, via the technique used by Callebaut (1971). This is also to examine the effect of the electrical conductivity on the instability of such model. In view of some practical applications in industries (e.g. the correction due to non-linear terms) extensions are considered. Moreover, the results were very rewarding theoretically also because several unexpected features turned up as well in the linear theory as in the non-linear one.

Keywords:
Magnetohydrodynamics – Stability – Compressible – Incompressible – Gas Cylinder – Liquid Cylinder–Streaming – Selfgravitating – Hollow Jet.

CHAPTER I

INTRODUCTION

In this chapter we discuss the concept of stability given by Chandrasekhar(1981). A brief discussion of the techniques which may be used in stability problems analysis is introduced here. Also we present the basic hydrodynamics and magnetohydrodynamics (MHD) equations which are essential for formulating any stability problem. Different boundary conditions at the fluid interfaces are explained. Finally, we review on some reported works which are the foundation of the present work.

I.1 Stability Concept

Here we follow Chandrasekhar (1981) for describing the stability concept. Suppose we have a hydrodynamic system which is in a stationary state, i.e. in a state in which none of the variables describing it is a function of time (whether it is initially at rest or streaming). Let x_1, x_2,………x_j be a set of parameters which define this system. These parameters may include geometrical parameters (such as the dimensions of the system); parameters characterizing the velocity of the fluid; parameters characterizing the forces acting on the system such as electromagnetic force, pressure gradient; and others. In considering the stability of such a system we seek to determine the reaction of the system upon small disturbances. If the system is disturbed and the disturbance gradually die down, in this case the system is stable with respect to the particular disturbance. On the other side, if the disturbance grows in amplitude in such a way that the system progressively departs from the initial state and never reverts to it, we say that it is unstable. It should be noticed that a system cannot be considered stable unless it is stable with respect to every possible disturbance to which it can be subjected. In other words, stability must imply that there exists no mode of disturbance for which it is unstable.

If all initial states are classified as stable or unstable according to the criteria stated, then in the space of parameters, x_1, x_2,……… x_j, the locus which separates the two classes of states defines the states of marginal stability of the system. By this definition, a marginal state is a state of neutral stability.

In studying the stability of a hydrodynamic or hydromagnetic problem, it is often convenient to suppose that all parameters of the system, save one, are kept constant while the chosen one is continuously varied. We shall then pass from stable to unstable states when the particular parameter we are varying takes a certain critical value. We may say that instability occurs at this value of the chosen parameter while all the others have their reassigned values.

The states of marginal stability can be one of two kinds corresponding to the two ways in which the amplitudes of a small disturbance can grow or be damped: they can grow (or be damped) periodically, or they can grow (or be damped) by oscillations of increasing (or decreasing) amplitude. In the former case, the transition from stability to instability takes place via a marginal state exhibiting a stationary pattern of motions. In the latter case, the transition takes place via a marginal state exhibiting oscillatory motions with a certain definite characteristic frequency.

1.2 Applications of Hydrodynamic and MHD Stability

There are many applications of hydrodynamic and MHD stability in several fields of science such as

Geophysics

The fluid of the core of the Earth and other planets is theorized to be a huge MHD dynamo that generates the Earth's magnetic field due to the motion of the liquid iron.

Astrophysics

MHD applies quite well to astrophysics since 99% of baryonic matter content of the universe is made of plasma, including stars, the interplanetary medium, nebulae and jets, stability of spiral arm of galaxy,…etc. Many astrophysical systems are not in local thermal equilibrium, and therefore require an additional kinematic treatment to describe all the phenomena within the system.

Engineering Applications

MHD and hydrodynamic stability has many forms in engineering sciences include oil and gas extraction process, gas and steam turbines, MHD power generation systems and magneto- flow meters,….etc.

1.3 Analysis in Terms of Normal Modes

There are several methods for solving the stability problems. Only some of them are mentioned here: the energy principle method, multiple time scales method, the variations principle method and the normal mode method. Of course, every method has its advantages and disadvantages. For example, in the variations principle method we are only able to say that the model is stable or not, while in the normal mode method we could determine exactly the unstable domains and the critical value x_c of the longitudinal dimensionless wavenumber. In our work, we'll use the normal mode technique for the perturbations analysis . In this section we present a brief discussion of this technique.

The mathematical treatment of a stability problem generally starts from an initial flow which represents a stationary state of the system. By assuming that the various physical variables describing the flow suffer infinitesimal increments, we first obtain the equations governing these increments. In obtaining these equations from the relevant equations of motion, we neglect all products and powers higher than the first of the increments and retain only terms which are linear in them. That is why we call it the linear stability theory in contrast to non-linear theory in which we need to consider them.

As we have mentioned, stability means stability with respect to all possible infinitesimal disturbances. So, for investigation of stability to be complete, it is necessary that the reaction of the system to all possible disturbances must be examined. In practice, this is accomplished by expressing an arbitrary disturbance as a superposition of certain basic possible modes and examining the stability of the system with respect to each of these modes.

1.4 Hydrodynamics and Magnetohydrodynamics Basic Equations

Magnetohydrodynamics is defined as the study of the macroscopic interaction of electrically-conducting fluids with a magnetic field. In studying the magnetohydrodynamics stability of any model, it is important to study the equations governing this model which can be classified into two groups of equations. The first group is the hydrodynamics equations (the equation of motion and the continuity equation) and the second group is the equations of Maxwell concerning the electrodynamic theory.

In this section, these equations are presented in their general form and also the special cases are considered.

1.4.1 Continuity Equation

Consider a specific mass of fluid whose volume is arbitrary chosen. If given fluid mass is followed as it flows, its size and shape will be observed to change but its mass will remain unchanged. This is the principle of mass conservation which applies to fluids in which no nuclear reactions are taking place. It should be noticed that the density of individual particles. may or may not be conserved, since the volume of a particle may change during a motion. The continuity equation, in its general form, is given in the form

$$\frac{d\rho}{dt} + \rho(\nabla \cdot \underline{u}) = 0 \qquad (1.1)$$

with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\underline{u} \cdot \nabla)$$

where ρ is the fluid mass density and $\underline{u}$ is the fluid velocity vector. When the fluid under consideration is incompressible, that is the volume of each particle remains constant during the time of motion. Hence, the density remains constant ($d\rho/dt$=0), then the continuity equation reduces to the form

$$\nabla \cdot \underline{u} = 0 \qquad (1.2)$$

For an irrotational motion, the velocity vector must be expressed as a gradient of a potential function, say ϕ. Therefore, the velocity vector becomes

$$\underline{u} = \nabla \phi \, . \tag{1.3}$$

Consequently, for incompressible-irrotational flow, we have

$$\nabla^2 \phi = 0 \, . \tag{1.4}$$

1.4.2 The Vector Equation of Motion

Consider Navier-Stokes' equation which for a viscous fluid may be written in the form:

$$\rho \frac{d\underline{u}}{dt} = -\nabla P + \underline{F} - (\rho \upsilon) \nabla \wedge (\nabla \wedge \underline{u}) \tag{1.5}$$

where ρ, $\underline{u}$, P are the fluid mass density, velocity vector and kinetic pressure, υ the kinematical coefficient of viscosity (while $\rho\upsilon$ called the dynamical coefficient of viscosity), and $\underline{F}$ the external acting force per unit volume of the fluid. This external force could be the self-gravitating force, electromagnetic force or any other external force acting on the fluid.

It should be mentioned here that for an incompressible fluid and making use of the continuity equation (1.2) with the vector relation

Navier – Stokes' equation is simplified to

$$\rho \frac{d\underline{u}}{dt} = -\nabla P + \underline{F} + (\rho \upsilon) \nabla^2 \underline{u} \tag{1.6}$$

1.4.3 Maxwell's Equations

The electrical conductivity of the fluid and the prevalence of magnetic fields contribute to effects of two kinds: first, by the motion of electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic fields contribute changes in the existing fields; and second, the fact that the fluid elements carrying currents transverse magnetic lines of force contributes to additional forces acting on the fluid elements. With displacement currents ignored, Maxwell's equations are:

$$\nabla \cdot \underline{H} = 0 \tag{1.7}$$

$$\nabla \wedge \underline{H} = 0 \tag{1.8}$$

$$\nabla \wedge \underline{E} = -\mu \frac{\partial \underline{H}}{\partial t} \tag{1.9}$$

where $\underline{E}$ and $\underline{H}$ are the intensities of the electric and the magnetic fields, and μ is the magnetic permeability.

If $\underline{u}$ is the fluid velocity vector, the experience electric field as measured by a stationary observer will not be $\underline{E}$, but it will be $\underline{E} + \mu(\underline{u} \wedge \underline{H})$ and consequently the current density $\underline{J}$ is given by

$$\underline{J} = \lambda' [\underline{E} + \mu(\underline{u} \wedge \underline{H})] \tag{1.10}$$

where λ' is the coefficient of electrical conductivity.

By using equations (1.8) and (1.10) for equation (1.9), the equation of the magnetic field could be obtained in the general form

$$\frac{d\underline{H}}{dt} = \nabla \wedge (\underline{u} \wedge \underline{H}) - \nabla \wedge (\eta \nabla \wedge \underline{H}) \tag{1.11}$$

where η is the resistivity coefficient.

Combining Maxwell equations with the equation of motion and the equation of continuity, we may conclude that when a fluid moves in the presence of a magnetic field $\underline{H}$ or alternatively a magnetic field penetrates a fluid, an electric current $\underline{J}$ is produced. Such current will interact with the original magnetic field $\underline{H}$ producing electromagnetic force μ ($\underline{J}$ ^ $\underline{H}$) influences on the fluid. This force has a very interesting goal and plays an important role in stabilizing or destabilizing the fluid models.

Therefore, the basic equations governing an incompressible, viscous and resistive fluid pervaded by a magnetic field are:

$$\rho\frac{d\underline{u}}{dt} = -\nabla P + \underline{F} + (\rho\upsilon)\nabla^2\underline{u} + \mu(\nabla\wedge\underline{H})\wedge\underline{H} \tag{1.12}$$

$$\nabla\cdot\underline{u} = 0 \tag{1.13}$$

$$\nabla\cdot\underline{H} = 0 \tag{1.14}$$

$$\frac{d\underline{H}}{dt} = \nabla\wedge(\underline{u}\wedge\underline{H}) - \nabla\wedge(\eta\nabla\wedge\underline{H}) \tag{1.15}$$

1.5 Boundary Conditions

Based on the normal mode technique, the basic equations governing the model under consideration must be solved for the unperturbed state and also for the perturbed state. These solutions must satisfy certain conditions across the boundaries of the model in order to determine the constants of integration of the differential equations of the fluid model. These conditions are known as boundary conditions and they may be some of the following.

1. The kinematic boundary conditions which states that the normal component of the velocity of the fluid must be continuous and simultaneously compatible with the velocity of the deformed interface.
2. The normal component of the magnetic field must be continuous across the boundary interface.
3. The continuity of the self gravitating potential and its derivative across the perturbed interfaces.
4. The compatibility condition that the total pressure must be continuous across the deformed surface.

1.6 On Reported Works

In this section we turn our attention to a large and important group of problems: the stability of jets and superposed fluids for its applications in science. The classical example of jets is the instability of water issuing from a nozzle as a cylindrical jet. The cause of this instability, as Rayleigh (1945) first showed, is the surface tension which makes the infinite cylinder an unstable figure of equilibrium and entails its breaking into separate pieces with a total surface area which is less than that of the original cylinder. Plateau (1873) also concluded that the model is absolutely unstable only to axisymmetric perturbations whose wavelength is longer than the circumference $2\pi R$ of the jet (radius R) and stable to all other perturbations.