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A STUDY OF SOME TYPES OF BETA DISTRIBUTION

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ABSTRACT

A STUDY OF SOME TYPES OF BETA DISTRIBUTION

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Abstract

In this thesis, we are concerned with some one of important types of the beta distributions. We study the bivariate form of Connor and Mosimann's generalized Dirichlet distribution. This distribution has many applications, including Bayesian statistics, contingency tables and statistical decision theory. We introduce a simple way for generating observations from this distribution. We derive method of moment's estimators of the parameters of the distribution. A simulation study is performed to compare between the method of moments estimators derived and the corresponding maximum likelihood estimators (MLEs) based on bias and mean square error (MSE). The method of moment's estimators showed satisfactory results. It is well known that the distributions of the differences, ratios and products of random variables are of great interest in statistics and have several applications, such as reliability engineering, industrial engineering and computer systems. We derive exact and approximate distributions of the differences, ratios and product of the components (X and Y, say) of the Connor and Mosimann's bivariate Dirichlet distribution. For the exact distributions we obtain some statistical characteristics, such as the moments and hazard rates. Using the distributions of the differences and the ratios, we obtain the stress- strength reliability function $P(X < Y)$. A simulation study is performed to compare the exact and the approximate distributions showing evidence that the proposed approximate distributions are quite robust. Also this robustness is detected graphically. Moreover, applications for the exact distributions and the approximate distributions are provided to some real data.

Key words

Connor and Mosimann's generalized Dirichlet distribution; Connor and Mosimann's bivariate Dirichlet distribution; Hypergeometric function; Method of moments estimators.

SUMMARY

The beta distribution is one of the distributions that are applicable in different aspects, such as rule of succession and task duration modeling. Examples of events that may be modeled by beta distribution include: the time taken to complete a task and the proportion of defective items in a shipment. The beta distributions are used extensively in Bayesian statistics, since beta distributions provide a family of conjugate prior distributions for binomial and geometric distributions.

Bivariate beta distributions are used in a wide variety of applications such as Bayesian statistics and reliability theory. They form part of the Dirichlet family of distributions, but have become an important family by itself. Many bivariate beta distributions have been derived out of applications, as extensions, or generalizations from other well-known bivariate beta distributions.

This thesis aims to study the Connor and Mosimann's bivariate Dirichlet distribution, where the variables of the distribution, say X and Y may be considered as proportions. This distribution has several applications in many areas, such as statistical decision theory, contingency tables, geochemistry and forensic sciences. Data comprising of proportions arise in most areas of sciences and engineering. Some examples of data set that involve proportions are: sand, silt, clay compositions of sediment samples and compositions of lavas.

Linear combinations, ratios and products of two random variables have attracted many researchers in the statistics literature due to its practical importance in the fields of reliability engineering, industrial engineering and computer systems. The random variables may be independent or dependent. In this thesis, we derive the exact distributions of the difference $W = X - Y$, ratio $T = X / Y$ and product $P = XY$ of two dependent random variables X and Y following the Connor and Mosimann's bivariate Dirichlet distribution. Some statistical characteristics of these distributions are studied, such as the moments and the hazard rate function. As the exact distributions of W, T and P derived have complex forms involving the Gauss hypergeometric and the Appell

function ,we deduce approximate distributions for these variables depending on beta type I and type II distributions.

The thesis consists of four chapters as follows:

Chapter I

Introduction and Preliminary

In Chapter I, we present the basic definitions and the preliminary results that are used throughout the thesis. In this chapter we present a brief literature review, concerning the point of research, including univariate and bivariate beta distributions, exact and approximate distributions of combinations of beta distributed variables.

Chapter II

Connor and Mosimann's Bivariate Dirichlet Distribution

In Chapter II, we study the Connor and Mosimann's bivariate Dirichlet distribution. We discuss some statistical properties of this distribution, such as product moments, covariance and correlation coefficient. In addition, we consider the marginal and the conditional densities. Estimation of the parameters of the distribution is discussed. Maximum Likelihood estimators (MLE) are presented and method of moments estimators are derived. A simple way for generating observations from the distribution is introduced. Finally a simulation study is performed to show the efficiency of the estimators.

Chapter III

Exact Distributions of Differences, Ratios and Product

In Chapter III, we derive the exact distributions of the difference W, ratio T and product P of two dependent random variables X and Y following the Connor and Mosimann's bivariate Dirichlet distribution. We study some statistical characteristics of these variables, such as the moments and hazard rates .Using the distributions of the differences and the ratios, we obtain the stress-strength probability $P(X < Y)$. Moreover

an application of the results to some real data is provided. The results of this chapter are published in "Journal of Statistics: Advances in Theory and Applications".

Chapter IV

Approximate Distributions of Differences, Ratios and Product

In Chapter IV, we derive the approximate distributions of the difference W, ratio T and product P of two dependent random variables X and Y following the Connor and Mosimann's bivariate Dirichlet distribution. A simulation study is performed to show evidence that the proposed approximate distributions are quite robust. The robustness is also verified graphically. Finally an application of the theoretical results to some real data is provided. The results of this chapter are submitted for publication in the Journal "Statistics".

CHAPTER (I)
INTRODUCTION AND PRELIMINARIES

1.1 Introduction

The beta distribution is a family of continuous probability distributions defined on the interval (0, 1) parameterized by two positive shape parameters, typically denoted by (α, β). This distribution is one of the most important distributions in probability theory and statistics. It has been applied to model the behavior of random variables limited to intervals of finite length in a wide variety of disciplines. For example, it has been used as a statistical description of Allele frequencies in population genetics (Balding and Nichols [1995]); time allocation in project management / control systems (Malcolm and et al [1958]); sunshine data(Sulaiman et al [1999]); variability of soil properties (Haskett et al [1995]); proportions of the minerals in rocks in stratigraphy (Gullco and Anderson [2009]); and heterogeneity in the probability of HIV transmission(Wiley et al [1989]).

In Bayesian inference, the beta distribution is the conjugate prior probability distribution for the Bernoulli, binomial and geometric parameters. For example, the beta distribution can be used in Bayesian analysis to describe initial knowledge concerning probability of success such as the probability that a space vehicle will successfully complete a specified mission. The beta distribution is a suitable model for the random behavior of percentages and proportions.

In this chapter, we present the basic definitions and the preliminary results that are used throughout the thesis. In Section (1.2) preliminary results on some special functions are present. In Section (1.3), we define some types of univarite beta distributions. Section (1.4) is devoted to present some types of bivariate beta distributions. We present, in Section (1.5), some methods of estimation and some estimators properties. In Section (1.6), we present the Kolmogorov-Smirnov goodness of fit test. Finally, we present a brief literature review, concerning the point of research, including univariate and bivariate beta distributions, exact and