بسم الله الرحمن الرحيم

شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

قَسَــم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها علي هذه الأفلام قد اعدت دون آية تغيرات

يجب أن

تحفظ هذه الأفلام بعيداً عن الغبار

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To be kept away from dust in dry cool place of 15 – 25c and relative humidity 20-40 %

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بالرسالة صفحات

لم ترد بالأصـــــل

FACULTY OF EDUCATION
AIN SHAMS UNIVERSITY

ON GENERALIZATIONS OF CS-MODULES

THESIS

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CONTENTS

SUMMARY

A submodule N of a module M has no proper essential extensions in M, if and only if there is another submodule N′ such that N is maximal with respect to $N \cap N' = 0$. In the literature, such submodules N are called closed, or complements. It is well known that, for any submodule N of M, there exists a closed submodule K of M such that N is essential in K, and K is called a closure of N in M.

A module with the property that every submodule has a unique closure (in the module) is called a UC-module ("UC" for "unique closure"). P. F. Smith [50] investigated UC-modules. In this thesis we give some different characterizations of UC-modules which are related to the properties of essential extensions.

A module with the property that "every closed submodule is a summand" is called a CS-module ("CS" for "closed are summands"). This property "CS" holds in particular if the module is (quasi) injective, or more generally (quasi) continuous.

A module M is called a continuous module if it is a CS-module and satisfies the following condition (C_2) :

for which mutually injectivity occurs for all their direct sum decompositions. Kamal called such modules DRI- modules, while Burgess and Raphael called them ads-modules.

Recently, D. V. Huynh, and B. J. Müller [24] studied rings for which direct sums of extending modules are extending.

The present thesis, which consists of five chapters, focuses on some important aspects of the theory of UC-modules, ads-modules, Co-semisimple modules, and Completely CS-modules.

The first chapter provides the preliminaries and some background results to be used in subsequent chapters. Such as, basic definitions of modules, Local summands, Indecomposaple decompositions, essential, small and closed submodules, some special classes of modules, the socle and the radical and projective and injective modules.

In the second chapter we investigate modules with the property that every submodule has a unique closure.

The aim of this chapter is to study the necessary and sufficient conditions for a module to be a *UC-module*. We display some important informations and results which help to understand, and to study the various characterizations of *UC-modules.* In fact Lemma (2.11) gives the necessary and sufficient condition for an essential extension of a *UC-module* to be a *UC-module.* Corollary (2.13) indicates when a module with essential socle is a *UC-module.*

Finally, we discuss some basic facts about the concept of R-closed submodules of a module M.

In the third chapter, modules with the absolute direct summand property are studied. These modules M are characterized by a certain property on their direct sums decompositions: If $M = A \oplus B$ and C is a complement of A in M, then $M = A \oplus C$. (We call such modules M ads-modules).

It is clear that every quasi-continuous module enjoys such property. Since indecomposable quasi-continuous module are uniforms, and since every indecomposable module is an ads-module, then our condition is strictly weaker than quasi-continuity.

Our main goal is to investigate the necessary and sufficient conditions for a module to be an ads-module. The

Some of the proofs we have given here are slightly different, and some others are easier and completely different, from the ones originally given by the othors.

Some of the results in chapters 2, 3, 4, and 5 are, in my openion, new results [(2.4), (2.11), (2.13), (3.8), (3.9), (4.2), (4.3), (4.9), (4.11), (4.12), (4.13), (4.14), (4.17), (4.18), (4.20), (4.25), (4.26), (4.32), (4.33), (4.39), (5.10), (5.14), (5.15), (5.24)].

All rings considered, have an identity, and are not necessary commutative. All items, such as Theorems, Propositions, Lemmas, Corollaries, and Definitions, are numbered consecutively, (m.n), where m indicates the chapter and n the place of the item within the chapter.