Ain Shams University
University College of Women
for Arts, Sciences and Education
Mathematics Department

THEORETICAL STUDY OF CERTAIN PROBLEMS OF SPECIAL FUNCTIONS AND FRACTIONAL CALCULUS OPERATORS

THESIS SUBMITTED

By

Ahmad Wedad Faraj

FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY (Ph.D.)
IN SCIENCE
(PURE MATHEMATICS)

SUPERVISORS

Prof. Dr. Safaa S. Bishay
Professor of Applied Mathematics
University College of Women for
Arts, Sciences and Education
Ain Shams University

Prof. Dr. Gamal F. Ismail
Professor of Pure Mathematics
University College of Women for
Arts, Sciences and Education
Ain Shams University

Prof. Dr. Tariq O. Salim
Professor of Applied Mathematics
College of Science
Al-Azhar University - Gaza

2015

بسم الله الرحمن الرحيم

﴿قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ﴾ [البقرة:32]

﴿وَمَا تَوْفِيقِي إِلَّا بِاللَّهِ عَلَيْهِ تَوَكَّلْتُ وَإِلَيْهِ أُنِيبُ﴾ [هود:88]

Before I begin, I want to start with thanking my Lord "Allah" who is always teaching me and guiding me.

Ain Shams University
University College of Women
for Arts, Sciences and Education
Mathematics Department

Ph.D. Thesis
(PURE MATHEMATICS)

Title of thesis

THEORETICAL STUDY OF CERTAIN PROBLEMS OF SPECIAL FUNCTIONS AND FRACTIONAL CALCULUS OPERATORS

By
Ahmad Wedad Faraj

Thesis supervisors

Prof. Dr. Safaa S. Bishay
Professor of Applied Mathematics
University College of Women for
Arts, Sciences and Education
Ain Shams University

Prof. Dr. Gamal F. Ismail
Professor of Pure Mathematics
University College of Women for
Arts, Sciences and Education
Ain Shams University

Prof. Dr. Tariq O. Salim
Professor of Applied Mathematics
College of Science
Al-Azhar University - Gaza

2015

Dedication

- To my father, who taught me that Math is an art in addition to a science.

- To my teacher, the martyr Fayez Badran.

  May Allah rest their souls.

- To my sons Areej, Wedad and Nedal.

Acknowledgment

I wish to express my gratitude and appreciation to **Dr. Safaa Sadek**, **Dr. Gamal Ismail** and **Dr. Tariq Salim** for their constructive and fruitful help throughout writing this dissertation.

Special thanks to my friend **Mustafa Shaheen** for his great work in printing and preparing this thesis.

Finally, my deep thanks to my family especially **Nasra, Nancy, Amal, Ashraf** and **Usama** who encouraged and pushed me forward to achieve the present work.

Abstract

Abstract

Ahmad Wedad Faraj. Theoretical Study of Certain Problems of Special Functions and Fractional Calculus Operators. Doctor of Philosophy in Science, Dissertation of Pure Mathematics, University College of Women for Arts, Sciences and Education, Ain Shams University.
The main purpose of this thesis is to introduce generalized forms of some special functions of fractional calculus that play an important role in solving fractional differential and integral equations.

This thesis is divided into five chapters

In chapter 1: The literature review of the problems under study is indicated, moreover the related definitions, preliminaries and formulas needed are stated.

In chapter 2: We review some results refer to the generalized Mittag-Leffler function and extend them to obtain new properties and formulas, also we discuss its relation to fractional calculus operators. We establish an integral operator called Weyl integral operator containing the generalized Mittag- Leffler function in its kernel and derive its various properties. A solution of fractional nonhomogeneous differential equations are investigated in terms of the generalized Mittag – Leffler function.

In chapter 3: We introduce a new generalized M-series and study its main properties including the relation with fractional calculus operators. We also establish an integral operator containing the generalized M-series in its kernel. At the end of this chapter we define a general class of polynomials associated with the generalized M– series and bring out several families of generating relations and finite summation formulas.

In chapter 4: We introduce a new generalized K_4- function and investigate its relation with the generalized Mittag – Leffler function and the generalized M-series. Furthermore we solve the general fractional kinetic equation in terms of the special functions by applying two different methods; Laplace transform operator method and differ – integral operator method.

In chapter 5: Concluding remarks of the thesis, comments and directions for further work are stated.

Keywords : Generalized Mittag- Leffler function, generalized M- Series, generalized K_4 - function, Riemann- Liouville fractional calculus operators, Weyl fractional integral and differential operators, generalized fractional kinetic equation, fractional differential equations, Laplace transform operator, general classes of polynomials.

Summary