Cairo University
Institute of Statistical Studies and Research
Department of Operations Research

A Comparison Between Fuzzy Approaches For Solving The Critical Path Problem

By

Soha Mohamed Mohamed Abd-Allah

A thesis submitted to the Institute of Statistical Studies and Research
Cairo University

In partial fulfillment of the requirements for the Degree of

Master of Science
In
Operations Research

Under the Supervision of

Prof. Dr. Hegazy Mohamed Zaher
Professor of Mathematical Statistics
Dept. of Statistics, Institute of Statistical Studies and Research
Cairo University, Egypt

Dr. Mohamed Bayoumi Ali
Associate Professor
Dept. of Decision Support, Faculty of Computers and Information
Cairo University, Egypt

Dr. Naglaa Ragaa Saied
Assistant Professor
Dept. of Operations Research, Institute of Statistical Studies and Research
Cairo University, Egypt

2011

Dedication

My mother & father
They were helping me from the beginning till the end

My brothers
They were supporting me from the beginning till the end

Contents

List of Abbreviations

AOA	Activity-On- Arrow
AON	Activity-On- Node
CPM	Critical path method
D_{ij}	Duration of activity(i ,j)
ES	Earliest Start times
FF	Free Float
FPNA	Fuzzy project network analysis
L-R	Left and Right shape function of the modal value
LC	Latest Completion times
MF	Membership function
PERT	Program Evaluation and Review Technique
TF	Total Float
$\hat{E}_j$	The fuzzy earliest time of event j
$\widetilde{EF}_{ij}$	The fuzzy earliest finishing of activity (i, j)
$\widetilde{TS}$	The fuzzy time of starting the project
$L\tilde{F}_{ij}$	The fuzzy latest finishing of activities (i, j)
$L\tilde{S}_{ij}$	The fuzzy latest starting of activities (i, j)
L-L	Left and Right shape function of the modal value are the same
ls and rs	Being the left and right spreads that are non-negative real numbers

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Acknowledgements

I would like to express my sincere appreciation to my advisors, Professor Hegazy Zaher, for his guidance, motivation and encouragement. Besides his support with an enormous amount of energy and time in directing my research, his professionalism is a great inspiration to me, I wish to thank Dr. Mohamed Bayoumi and Dr Naglaa Ragaa Saied for their advices and directions.

I would like to thank Eng. Ayman M.M.Abd-Allah ElFaramawi, MAKRO CASH AND CARRY Salam city branch consultant's project manager for providing me all construction data of MAKRO store. Also I would like to thank Mr. Mohamed Fathi Goda Elsantawy for his great effort in helping me in the Matlab processing tasks.

Finally, I would also like to thank my parents, my brothers and my friends for their invaluable support and encouragement.

Summary

Three approaches are introduced, the first is a previous study (Chen’s approach) and the other two are suggested (the first proposed approach and the second proposed approach), to solve the problem of critical path in a project network with fuzzy activity times using fuzzy numbers. The two suggested approaches are explained by applying them to the example of Chen and Hsueh’s approach. The three approaches are applied to a real case: Makro Mall in Al-Salam City for wholesales using the MATLAB software. These approaches help the decision maker to determine the critical path, total duration time, critical activities and degree of criticality for all activities.

A comparison is made between the three approaches to determine the best approach for the practical reality. It was found that the second proposed approach is the best, it is characterized by its ease of calculations, also it gives more realistic results for total duration time and it provides the decision maker with better information that helps him to make the suitable decision.

The thesis consists of four chapters:

Chapter 1: It gives an introduction, explaining the outline of the thesis, research objectives, contributions of this work and it is made for the previous related work done in fuzzy project management field.

Chapter 2: It includes explanations of the terms, phrases, and concepts used to conduct the research investigation. The definitions and concepts are organized into four sections; Project Management, Fuzzy Set Theory, Fuzzy number, and Time Representation.

Chapter 3: Two previously developed fuzzy approaches are explained (Chen’s approach and Chen & Hsueh’s approach); also two fuzzy approaches are proposed and applied to the same problem that the former two approaches are applied to. Chen and Hsueh's approach has been developed to obtain the first proposed approach. All of approaches are compared on an existing example.

Chapter 4: It presents the Chen’s approach and the two proposed approaches. All approaches are applied to a real construction case study and a comparison is made between results to determine the best approach. MATLAB software version 7.14 has been employed to develop a code to solve all three approaches. Also, it contains conclusions and future research.

Chapter 1

Chapter 1
Introduction

1.1 Background

Project management is an important technique to help a project manager to plan, execute, and control his projects [17].In recent years; the range of project management applications has greatly expanded. Project management concerns the scheduling and control of activities (tasks) in such a way that the project can be completed in as little time as possible [11]. To ensure the project's success, the project management team must identify the stakeholders, determine their needs and expectations, and manage those needs and expectations [11].

The most commonly used project management two techniques are Critical Path Management (CPM), and Project Evaluation and Review Technique (PERT) [11].

By using CPM or PERT, managers are able to obtain [11, 23]:

1. A graphical display of project activities (tasks).
2. An estimate of how long the project will take.
3. An indication of which activities are the most critical for timely project completion.
4. An indication of how long any task can be delayed without delaying the project.

When the activity times in the project are deterministic and known, critical path method (CPM) has been demonstrated to be a useful tool in managing projects in an efficient manner to meet this challenge [23]. However, there are many cases where the activity times may not be presented in a precise manner. To deal quantitatively with imprecise data, the program evaluation and review technique (PERT) [12] based on the probability theory can be employed. An alternative way to deal with imprecise data is to employ the concept of fuzziness, whereby the vague activity times can be represented by fuzzy sets. In this case activity times are represented by fuzzy number [12].

Several studies have investigated the case where activity times in a project are approximately known and are more suitably represented by fuzzy sets rather than crisp numbers [12].Fuzzy set theory is developed as a tool to handle uncertainties that are nonstatistical in nature (zadeh 1965). The theory is designed particularly to model imprecise linguistic terms such as “approximately”, “about”, or “long”, etc. normally found in human expert statements [35].

1.2 Research Objectives

The main goal of this thesis is to apply three fuzzy approaches to a real-life construction case study. In order to attain this goal many sub-goals were set, the first goal is to find the critical path in a project network with fuzzy activity times and then solve it using the three approaches, the second goal is to compare the resulting solutions in order to determine the best, the simplest, and the most realistic, to provide the decision maker with better information that will help him in making the suitable decision, the third goal is to present a comprehensive survey that focuses on the fuzzy project management proposals introduced to this research area.

1.3 Historical Review

Prade, H. (1979) [42], he applied fuzzy set theory to the development of an academic quarter schedule at a French school. When data are not precisely known, fuzzy set theory is shown to be relevant to the exact nature of the problem rather than probabilistic PERT or CPM. The aim of his work is to show how and when it is possible to use fuzzy concepts in a real world scheduling problem. An overview of a fuzzy modification to the classic Ford solution algorithm was presented along with a 17 node network representation for the academic scheduling problem. Calculations were demonstrated for a small portion of the overall scheduling problem.

Chanas, S. and Kamburowski, J. (1981) [9], in this paper, they argued the need for an improved version of PERT due to three circumstances: (i) the subjectivities of activity time estimates; (ii) the lack of repeatability in activity duration times; and (iii) calculation difficulties associated with using probabilistic methods. A fuzzy version of PERT (FPERT) was presented in which activity times were represented by triangular fuzzy numbers.

Kaufmann, A. and Gupta, M. M. (1988) [25], they devoted a chapter of their book to the critical path method in which activity times are represented by triangular fuzzy numbers. A six step procedure was summarized for developing activity estimates, determining activity float times, and identifying the critical path.

McCahon, C. and Lee, E. (1988) [38], they noted that PERT is best suited for project network applications when past experience exists to allow the adoption of the beta distribution for activity duration times and when the network contains approximately 30 or more activities. When activity times are vague, the project network should be modeled with fuzzy components. A detailed example demonstrates modeling and solving an eight activity project network when activity durations are represented as triangular fuzzy numbers.

Buckley, J. J. (1989) [5], this proposal provided detailed definitions of the possibility distributions and solution algorithm required for using fuzzy PERT. A ten activity project network example, in which activity durations were described by triangular fuzzy numbers, was used to demonstrate the development of the possibility distribution for the project duration. Possibility distributions for float, earliest start, and latest start times were defined, but not determined, due to their complexity.

Lootsma, F. A. (1989) [33], in this proposal, he identified that the human judgment plays a dominant role in PERT due to the estimation of activity durations and the requirement that the resulting plan be tight. This aspect of PERT exposes the conflict between normative and descriptive modeling approaches. Lootsma argues that vagueness is not properly modeled by probability theory, and rejects the use of stochastic models in PERT planning when activity durations are estimated by human experts. Despite some limitations inherent in the theory of fuzzy sets, fuzzy PERT, in many respects, is more realistic than stochastic PERT.

DePorter, E. L. and Ellis, K. P. (1990) [14], they presented a project crashing model using fuzzy linear programming. Minimizing project completion time and project cost are highly sought yet conflicting project objectives. Linear programming allows the optimization of one objective (cost or time). Goal programming allows consideration of both time and cost objectives in the optimization scheme. When environmental factors present additional vagueness, fuzzy linear programming should be used. Linear programming, goal programming and fuzzy linear programming are