The Relative Motion of Two Artificial Satellites

Thesis by

Wael Mohamed Yousef

Submitted to

Mathematics Department - Faculty of Science - Ain Shams University
In Partial Fulfillment of the Requirements for the
Degree of Doctor Philosophy

Supervision Committee

Prof. Dr. Sami Mohamed El-Shaboury
Department of Mathematics
Faculty of Science
Cairo University

Prof. Dr. Moustafa Kamal
Department of Astronomy
Faculty of Science
Cairo University

Ass. Prof. Dr. Medhat Ammar
Department of Mathematics
Faculty of Science
Helwan University

APPROVAL SHEET

Title of the Ph. D. Thesis:

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Name of the candidate: **Wael Mohamed Yousef**

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Prof. Dr. Moustafa Kamal
Department of Astronomy
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Cairo University

Ass. Prof. Dr. Medhat Ammar
Department of Mathematics
Faculty of Science
Helwan University

Discussion Committee

Prof. Dr. Sami Mohamed El-Shaboury
Department of Mathematics
Faculty of Science
Cairo University

Prof. Dr. Mohamed Radwan
Department of Astronomy
Faculty of Science
Cairo University

Prof. Dr. Inal Adham Hasan
Department of Astronomy
Faculty of Science
Al-Azhar University

ACKNOWLEDGMENTS

*Firstly, I must kneel humbly forever to **ALLAH** thanking **HIM** for showing me the right path, without **HIS** help my efforts would have gone astray.*

Then I am expressing my deepest gratitude to my great parents, my wife, her father, my brothers and sister for their support and prayers.

I would like to express my appreciation to the Associate Professor Dr. Medhat Kamal Ammar, for his continuous supervision as well as his running discussions throughout this work.

I am greatly indebted to Professor Dr. Sami Mohamed ElShaboury for his valuable advice. Really, I owe him a lot that words is poor to express.

*Also, I am asking **ALLAH** to have mercy on Professor Dr. Moustafa Kamal who has credited with selecting the title and starting this thesis.*

ABSTRACT

The present work deals with the study of relative motion of a deputy (follower) satellite with respect to a chief (leader) satellite that both orbit around the same central body (the Earth) in elliptical or circular orbits. The perturbations due to Earth oblateness (up to second order in terms of the oblateness coefficient of the Earth) and atmospheric drag were taken into account in our study. The relative equations of motion in the case of circular orbits were represented using the time as the independent variable. While in the case of elliptical orbits, the true anomaly of the chief's satellite is used as the independent variable of the relative equations of motion in order to simplify the system of differential equations that obtained to solve the problem.

SUMMARY

The present work deals with the study of relative motion of a deputy (follower) satellite with respect to a chief (leader) satellite that both orbit around the same central body (the Earth) in elliptical or circular orbits. The perturbations due to Earth oblateness (up to second order in terms of the oblateness coefficient of the Earth) and atmospheric drag were taken into account in our study. The relative equations of motion in the case of circular orbits were represented using the time as the independent variable. While in the case of elliptical orbits, the true anomaly of the chief's satellite is used as the independent variable of the relative equations of motion in order to simplify the system of differential equations that obtained to solve the problem.

The thesis consists of five chapters:

In chapter one, the relative motion problem of satellites is introduced, stating the problem statement, applications and the previous contributions to this problem. Then, the thesis outlines had been mentioned.

In the second chapter, a review of several basic concepts from orbital mechanics and mathematics is presented. This review is brief and only includes what is necessary to give a complete picture of this problem. It starts with introducing the definition and transformation between coordinate axes used in the thesis, then an overview of orbital elements and different types of time that used in orbital mechanics.

The third chapter had been specified to discuss separately the perturbations due to Earth oblateness and air drag, which in turn affect the satellite orbital motion. For that end, the Lagrangian and Gaussian variation of parameters techniques are introduced. Also, the rate of change of the orbital elements due to theses perturbations are derived. Then, we end this chapter by talking about the satellite life time.

In the fourth chapter, the equations of relative motion of a deputy (follower) satellite with respect to a chief (leader) satellite, that both orbiting the Earth are presented. Here, we assume that there is no any external forces act on the satellites except the central attraction force of the symmetrical spherical model of the Earth, which is known as the unperturbed case. Then, the derived equations of motion had been solved using two approaches, which are the Laplace transformation and the state transition matrix. In each approach, we consider the case of circular orbits and the general case of elliptical orbits. At the end of this chapter, we apply numerical simulations on our solutions.

In chapter five, which is the final chapter, we study the relative equations of motion again after adding separately the effect of perturbations due to Earth oblateness and air drag, showing how these equations and their solutions had been affected by such perturbations. But, we follow the Laplace transformation approach only in this chapter. Then, we apply the solutions on five numerical examples, considering the circular and elliptical orbits for each perturbation type. Finally, we talked about some important conclusions in response to this study.

Contents

CHAPTER (1) INTRODUCTION

1.1. BACK GROUND

The analysis and modelling of the relative motion between two satellites is of immediate interest to be able to design and develop multiple satellite missions such as satellite constellations and formations.

In the study of the relative motion dynamics of two or more satellites, one of the satellites is generally called the Chief (or Target satellite), and the others are called the Deputies (or follower satellites). Consequently, interest arises in the relative motion of the Deputies with respect to the Chief, with a reference frame centered on the Chief. The simplest set of differential equations that model the relative motion between a Deputy and Chief satellite are the Hill-Clohessy-Wiltshire (HCW) equations [1, 2]. These are a set of three, second-order linear differential equations. These equations, while easy to analyze, do not accurately describe the system dynamics, due to the assumptions of a circular reference orbit, linearized differential gravitational attraction and spherical Earth.

Some of the earlier works on this subject were motivated by a need to solve the satellite rendezvous problem for near-circular orbits.

In this thesis we derived the equations of motion of a single satellite under the influence of the gravitational oblateness arising from the aspherical property of the Earth, and the effect of air drag on the orbital elements of the satellite in low Earth orbit. We then use the Variation of Parameters (VOP) in the form of Lagrange for the effect of oblateness, and that of Gauss, for the effect of drag force to derive the time rates of change for the corresponding orbital elements in each case. The relative motion of two satellites on neighboring elliptic orbits were modelled using two reference coordinate systems, the Earth Centered Inertial coordinate system (ECI) and the Local Vertical Local Horizontal coordinate system (LVLH). Then we used Laplace transform method to solve the system of the equations of motion. The resulting expressions for relative position and velocity were plotted and checked for certain cases to compare with the other results arising from other researchers.

1.2. APPLICATIONS

Relative motion equations have seen several different application areas in the history of orbital mechanics. The first use was by Hill [1] in the late 19th century who was studying the motion of the Moon. His goal was to construct a more mathematically sound means of developing tables of lunar motion, which, at the time, were based on "practical astronomy rather than of mathematics" in his words.

The first aerospace applications were in the area of intercept and rendezvous mechanics during the late 1950's and continuing today. The intercept problem is one in which a chase vehicle is forced in such a way that its path intersects the path of a target point (which may be occupied by another vehicle) at a specified time. The rendezvous problem further insists that the relative velocity of the two spacecraft be driven to zero at the time of intersection so that a docking procedure or other such activities may be conducted. This problem was studied by Clohessy and Wiltshire [2] in the interest of developing a guidance scheme for the rendezvous problem assuming that the target vehicle was in a circular orbit. This target satellite was to be a control center issuing relative position and velocity data to the slave satellites, which then used an on-board propulsion system to carry out the rendezvous and docking maneuver.

Anthony and Sasaki [3] further studied the rendezvous problem after developing a higher order approximation of the relative motion equations. Using these new equations, the velocity impulse requirements for the rendezvous maneuver were developed and an analysis of the miss distance due to the approximation was conducted.

Kelly [4] developed an optimal solution to the two impulse rendezvous problem using relative motion equations and also includes the effects of eccentric orbits and gravity perturbations. A nonlinear model of relative motion was also given, but an analytical solution was not developed and so required numerical integration to solve the problem.

More recent applications of relative motion concepts are in the area of satellite formations. Satellite formations are of great interest, because it is thought that large numbers of simple, low efficiency satellites working in a cohesive fashion can produce better results than a single, high performance satellite. Such formations can also achieve a