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 **AIN-SHAMS UNIVERSITY**

College for Women

**Solution of some Torsion Problems
for Nonhomogeneous Elastic Bodies
of Revolution**

A Thesis presented by

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For

THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS

Under the supervision of

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To

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the 1990s, the number of people in the world who are undernourished has increased from 600 million to 800 million.

There are a number of reasons for this. First, the world population has increased by 1.5 billion in the last 20 years. Second, the world population is ageing. The number of people aged 65 and over has increased from 200 million in 1980 to 400 million in 1999. Third, the world population is becoming more urban. The number of people living in cities has increased from 1 billion in 1980 to 2 billion in 1999. Fourth, the world population is becoming more mobile. The number of people who have moved from one country to another has increased from 100 million in 1980 to 200 million in 1999.

These four factors have all contributed to the increase in the number of people who are undernourished. In addition, there are a number of other factors that have contributed to this increase. These include the following:

- The increase in the number of people who are living in poverty.
- The increase in the number of people who are living in urban areas.
- The increase in the number of people who are living in mobile communities.

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ABSTRACT

The Thesis is devoted to the study of the influence of non-homogeneity on the stressed state of elastic bodies subjected to torsion, in particular bodies of revolution under mixed boundary conditions. In some cases, the body may include a crack. We obtain analytical expressions for some quantities of physical importance such as the applied torque and the stress intensity factor in terms of auxiliary functions to be determined from the mathematical solution. Each problem leads to one or two coupled Fredholm integral equations of the second kind, the solutions of which are obtained either numerically or by using Poincaré expansions in a small parameter determined from the geometry of the body.

We analyze the obtained results for the numerical values for the stress intensity factor at the points of discontinuity of the boundary conditions. We also compare our results with those known in the literature.

KEYWORDS

Elasticity, Axisymmetric torsional problems, Bodies of revolution, Nonhomogeneous elastic bodies, Half-space with surface constraint, Crack problems, Elastic layer.

