AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
MECHANICAL POWER ENGINEERING

The laminar Forced Heat and Momentum Transfer in Developing flow in the Entrance region for Isothermal and Iso-Heat Flux walls of circular and flat rectangular ducts

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering
(Mechanical Power Engineering)

by

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STATEMENT

This thesis is submitted as a partial fulfillment of Master of Science in Mechanical Engineering, Faculty of Engineering, Ain shams University.

The author carried out the work included in this thesis, and no part of it has been submitted for a degree or a qualification at any other scientific entity.

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Abstract

The velocity and the temperature profiles development from initial uniform distribution at the entrance of a channel towards the fully developed profiles downstream causes a desirable heat transfer enhancement associated with an increase of the pressure drop due to the entrance effect which requires more power to overcome the increased power consumption. The use of short flow passages became very important in the design of the compact heat exchangers to have higher heat transfer rates over the whole flow passage length thus decreasing the dimensions of the heat exchanger.

Laminar heat transfer and flow development in the entrance regions of circular tubes and parallel-plate ducts is of considerable practical significance, and not surprisingly, there exists a large number of references in the literature on this topic, especially for incompressible laminar flow .Most of the investigations are based on the boundary layer model which approximates the actual flow case only at high Reynolds numbers on a portion of the developing length, away from the entrance, whose length depends on the value of the Reynolds and Prandtl numbers. In recent years some numerical solutions of the complete Navier-Stokes and the energy equations have been carried out numerically with finite difference, finite element and finite volume methods.

In this work numerical solutions of the full Navier Stokes equations with no approximations in the form of the vorticity and stream function, the Poisson type pressure equation and the full energy equations for parallel plate ducts and circular tubes are carried on, presenting the hydrodynamic and thermal developing lengths, the pressure drop, the local and the average friction factors , the velocity , the pressure and the temperature as well as the local and average Nusslet number distributions showing the effect of the axial diffusion of momentum and heat.

The laminar incompressible fluid flow field in the entrance region of parallel plate ducts and circular tubes is solved using the transient approach procedure. The unsteady state two-dimensional Navier Stokes equations are transformed into the transient vorticity transport equation. The stream function and the vorticity relationship, which is a Poisson type equation, is used to calculate the stream function. The velocity components U and V are obtained from the stream function field using their relation with the stream function. The boundary conditions for the two velocity components U and V are set to satisfy the nonslip condition at the walls, the irrotational flow condition at the entrance for which both of the velocity components U and V at the entrance are uniform, the symmetry with respect to the axes of the conduits and the fully developed flow condition beyond the developing lengths. The vorticity transport and the stream function-vorticity equations are cast in finite difference forms which satisfy both of the governing differential equations and the boundary conditions.

The chosen finite-difference scheme is validated through the comparison of the results obtained by it for the natural convection in a two-dimensional square cavity with a previously published solution for this case obtained using a series expansion of orthogonal functions for different Rayleigh numbers.

The pressure drop is obtained from the axial momentum equation as usually done by various procedures in boundary layer solutions from the solution of the core region but by applying Newton's second law of motion on the whole flow cross section. It is also obtained by solving the Poisson type pressure equation whose finite difference form represented the differential governing equation in the interior of the flow domain. At the walls and the boundaries the pressure Poisson type equation is cast in a finite difference form that satisfies the differential governing equation and the corresponding boundary conditions.

The laminar heat transfer in the entrance region of parallel plate ducts and circular tubes is investigated for the thermal boundary conditions of constant wall temperature CWT and constant wall heat flux CHF, using finite difference form of the two-dimensional energy equation in a dimensionless form neglecting the viscous dissipation effects with no heat generation. The method of the solution is the same as that used in the solution of the vorticity transport equation. The two cases are treated for Reynolds numbers from 100 to 2300 and Prandtl numbers 0.7, 1, 2, 5, 7 and 10 expressing different types of fluids.

The results of the pressure drop obtained by the above described two procedures are compared. Such a comparison has not been done before. The results show that the pressure drop, the Fanning and the apparent friction factors calculated from the pressure distribution obtained from the Poisson type pressure equation at the same axial distance from the entrance is higher than that obtained from the pressure drop calculated from the axial momentum equation. This difference is higher at the lower Reynolds numbers .In all cases the values of those parameters obtained by the two procedures become close as the flow approaches the hydrodynamic developing lengths.

The calculated values of the pressure drop, the average values of each of the fanning and the apparent friction factors calculated according to the present method taking into account the axial momentum diffusion term at any value of the dimensionless axial distance $X^{+} = x/deRe_{de}$ depends upon the value of the Reynolds number and differ considerably from those calculated from the boundary layer momentum equation solutions which show that these parameters depend only on the value of the dimensionless distance $X^{+} = x/deRe_{de}$and bear no relationship to the value of the Reynolds number.

The velocity distribution in the transverse direction undergoes overshoots. The encountered overshoots in the transverse distribution of the axial velocity component, the transverse pressure gradient and the adverse pressure gradient at the entrance next to the wall are presented and explained

Hydrodynamic developing lengths for each of the local Fanning, the apparent friction factors, the centerline velocity and the velocity field are obtained. The significance of each of them is examined. The calculations revealed that the spatial grid size has an important effect on the "developing length- Reynolds number" relationship as the developing length decreases with the increase of the Reynolds number when large spatial increments ΔX are used, while this relationship shows an opposite trend when small grid sizes are used. Empirical equations are developed representing these developing lengths.

The average Nusselt number $\overline{Nu_x}$ in most of the recent and old publications for some of which reference is made in the body of this work, are usually calculated from the value of the local Nusselt number by a direct integration method. This method is valid only for the case in which the difference between the local conduit wall temperature and the fluid local bulk temperature is constant and does not vary with the axial distance, which is not the case in the entrance region due to the thermal axial diffusion, and thus this procedure violates Netwon's law of cooling.

The proper procedure of obtaining both of the average Nusselt number, which satisfies Newton's law of cooling, and the average wall-fluid bulk temperature difference is discussed.

Similar to the case of the friction factors, the values of each of the local and average Nusselt number, calculated by the present method vary also considerably from those calculated from the energy boundary layer equation at any value of the dimensionless axial distance $X^* = x/dePrRe_{de}$ and depend upon the values of each of the Reynolds number and the Prandtl number.

The calculated values of the local Nusselt number Nu_x in this work agrees quite well with those recently obtained from the solution of the Navier-Stokes and the energy equations by finite differences taking into consideration the axial momentum and thermal diffusion terms. As should be the case, the calculated values of the average Nusselt number $\overline{Nu_x}$ calculated by the present method show intolerable difference from that obtained by the direct integration of the local Nusselt number Nu_x values that is calculated through taking the axial momentum and thermal diffusion terms. The values of the average Nusselt number $\overline{Nu_x}$ calculated by the present method are higher.

The axial distributions of the local and average Nusselt numbers are presented. The values of the local and average fluid bulk temperature for CWT and CHF as well as those of the wall for the CHF are developed. These parameters are prerequisites for the calculation of the convective heat transfer coefficients, heat transfer rates and fluid local and average bulk temperatures without recourse to tedious trial and error procedures. The thermal development lengths based on the local Nusselt number and the temperature profiles are determined. The relative importance of each of them is discussed and empirical equations are developed representing these developing lengths.

Abstract

The singularity in the velocity and temperature fields at the leading edge of the flow conduits does not allow the direct determination of the local friction factor or the local Nusselt number by any numerical method there. These values are prerequisites for the calculation of the correct average values of these parameters along the whole length of the wall. The previous work ignores this fact and makes no reference to the procedure of handling this problem. The necessary formula for the calculation of these parameters is derived. Furthermore the distribution of these parameters in the very close proximity to the leading edge from x=0 to small distances, $x^+ = (x/de, Re_{de})$ or $x^* = (x/de, Re_{de} Pr)$, are given in enlarged scale. The results offer the parameters in these regions where their values are the highest and no similar results for this region are available in literature even in the recent solutions which considered the axial momentum and heat diffusion terms.

Key words : Heat Transfer, Pressure drop , entrance region , Nusselt number, Friction factor.

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