Periodically Autoregression Processes and Some of Its Aspects

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by

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Dedication

To

My Parents Ibraheem and Zainab

My Brother Loay

My Sisters Marwa, Ikhlas, Sarah and Hadeel

Acknowledgments

I would like to express my sincere thanks and gratitude to Almighty for his blessings and keeping me.

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Contents

Abstract

This thesis is interested with some characteristics of a class periodically correlated time series models. We studied both periodically correlated time series models and multivariate models, and we discussed in this work many representations of periodically correlated models.

Periodic autoregressive moving average PARMA process extend the classical autoregressive moving average ARMA process by allowing the parameters to vary with seasons. Model identification is to establish an identification of a possible model based on an available realization which is the first step for fitting a model to time series data, i.e., determining the type of the model with appropriate orders. The periodic autocorrelation function (PeACF) and the periodic partial autocorrelation function (PePACF) serve as useful indicators of the correlation or of the dependence between the values of the series so that they play an important role in model identification. The identification is based on the cut-off property of the periodic autocorrelation function (PeACF) and the periodic partial autocorrelation function (PePACF). We modified an explicit expression for the asymptotic variance of the sample PeACF to be used in establishing its bands. Therefore, we used the structure of the periodic autocorrelation function which depends directly to the variance and we applied some simulated examples with program R which agrees well with the theoretical results.

The inverse of invertible standard multi-companion matrices will be derived and introduced as a new technique for generation of periodic autoregression models to get the desired spectrum and extract the parameters of the model from it when the information of the standard multi-companion matrices is not enough for the extracting of the parameters of the model.

We found an explicit expressions for the generalized eigenvectors of the inverse of invertible standard multi-companion matrices such that each generalized eigenvector depends on the corresponding eigenvalue therefore we obtained a parameterization of the inverse of invertible standard multi-companion matrix through the eigenvalues and these additional

quantities. The results can be applied to statistical estimation, simulation and theoretical studies of periodically correlated and multivariate time series in both discrete- and continuous-time series.

We gave a method for generation of periodically correlated and multivariate ARMA models whose dynamic characteristics are partially or fully specified in terms of spectral poles and zeroes or their equivalents in the form of eigen-(values/vectors) of associated model matrices by the inverse of invertible standard multi-companion(IISMC) when the information of the standard multi-companion matrices is not enough for the extracting of the parameters of the model. Our method is based on the spectral decomposition of inverse of invertible standard multi-companion(IISMC) matrices and their factorization into products of companion matrices, and we compared the gotten results from a real data with the last papers reached, and we found that our technique is better and consistent. Generated models are need in simulation but may also be used in estimation, some times to set sensible initial values of parameters for non-linear optimization.

Parameters and Abbreviations

Parameters

μ_t : expectation of a time series

σ_ϵ^2 : variance of a time series

$\gamma_X(s,t)$: autocovariance function of a time series

$\rho_X(h)$: autocorrelation function of a time series

ϕ_p : autoregressive parameter in a AR(p)model

θ_q : moving average parameter in a MA(q)model

$\boldsymbol{\mu}_t$: mean vector of a multivariate time series

$\Gamma(t+h,t)$: covariance matrix of a multivariate time series

$\phi_{s,i}$: periodic autoregressive parameter in a PAR(p)model for

$s = 0, ..., N-1$ and $i = 1, ..., p, \quad p = \max\{p_1, ..., p_d\}$

Φ_i : $m \times m$ matrix of periodic autoregressive parameters in a PAR(p)

model for $i = 1, ..., p, \quad p = \max\{p_1, ..., p_d\}$

$\theta_{s,i}$: periodic moving average parameter in a PMA(q)model for

$s = 0, ..., N-1$ and $i = 1, ..., q, \quad q = \max\{q_1, ..., q_d\}$

Θ_i : $m \times m$ matrix of periodic moving average parameters in a PMA(p)

model for $i = 1, ..., q, \quad q = \max\{q_1, ..., q_d\}$