Ain Shams University
Faculty of Science
Department of Mathematics

THE TRANSFER OF SOME ALGEBRAIC PROPERTIES BETWEEN RINGS AND SOME OF THEIR EXTENSIONS

A Thesis

Submitted to Department of Mathematics, Faculty of Science,
Ain Shams University for the Degree of Doctor of Philosophy
(Ph. D.) in Pure Mathematics

By

Hanan Abd-Elmalk Sayed Mohamed

Supervisors

Prof. Dr. Abdel-Aziz El-Azab Radwan
Professor of Pure Mathematics,
Department of Mathematics, Faculty of Science,
Ain Shams University

Prof. Dr. Refaat Mohamed Salem
Professor of Pure Mathematics,
Department of Mathematics, Faculty of Science,
Al-Azhar University

Dr. Mohamed Ahmed Farahat
Lecturer of Pure Mathematics,
Department of Mathematics, Faculty of Science,
Al-Azhar University

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وقل رب زدني علما


Dedicated
to
My Parents
and
Teachers

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Department of Mathematics

Ph. D. Thesis (Pure Mathematics)

Title of Thesis

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Lecturer of Pure Mathematics,
Department of Mathematics, Faculty of Science,
Al-Azhar University

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Hanan Abd-Elmalk

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SUMMARY

Summary

The study of the relation between rings and both their overrings (extensions) and subrings is a part and parcel of ring theory. The development of the formal study of ring theory has been guided by a huge number of different ring extensions; introduced and investigated for a variety of reasons.

The question of when do certain properties transfer from any ring R to its many types of extensions and vice versa has also been of interest to many algebraists for a long time. A similar question between a module and an overmodule has been pursued in module theory. These questions have been important topics of research and have been crucial in the development of algebra especially of ring and module theory.

It appears that the research work on the wide varieties of extensions is spread throughout the literature in disparate research papers.

The issue of ascertaining how various ring-theoretic concepts behave under certain types of change of rings, such as subrings and ring extensions has always been of fundamental interest among ring theorists. Therefore, the motivation of this thesis is to study the transfer of some algebraic properties between the base ring or module and some of their extensions.