#### **PREFACE**

The design incorrect manufacturing techniques are obvious reasons of the low reliability. Some manufactures hesitate to invest more money on an improved design and modern techniques of manufacturing and testing. Improper selection of materials is another cause for poor design. Components equipments do not operate in the same manner in all conditions. A complete knowledge of their characteristics, applications, limitations will avoid their misuse and minimize the occurrence of failures.

Accordingly, the main purpose of this thesis is to maximize the reliability of the system, so we study some different redundant systems. The system considered may be consisted of similar redundant units. The systems may have different types of operation (repair only, repair, replaced and preventive maintenance), the systems may have different types of failures (human errors, partial hardware failure, common-cause failure and total hardware failure). Using both the modern methods and the computer program to compute the several measures of reliability are obtained such as mean time to system failure, steady-state availability, and variance of time to failure, probability that the repairman is busy.

This thesis consists of seven chapters.

This first chapter presents some definitions, basic characteristics and some concepts for the reliability theory. Also the reliability function, the hazard rate, the cumulative hazard function and the mean residual life function, combinatorial aspects are defined, The Laplace transforms,

system of linear first orders differential equations, standby redundant system and the Markov models are defined. Also we taking some examples and graphs to showing the basic characteristics.

The second chapter presents a stochastic analysis of a dissimilar two-unit parallel system with preventive maintenance and commoncause failure. The system goes for preventive maintenance at random epochs.

A bivariate extension of the exponential distribution is taken for the lifetimes of two components. The failure, replacement and maintenance time's rates are assumed to the constants for the units. Using the theory of regenerative and Markov-renewal processes several measures of reliability are obtained. Certain important results have been derived as particular cases. By using computer programs some numerical results of steady-state availability, mean-time to system failure and profit analysis are calculated. The plots for system reliability, system mean time to failure and system availability are drawn for various parametric values. The effect of preventive maintenance on the system performance is shown graphically. The results obtained by Coel and Preeti (1992) are derived from the present chapter as particular cases. The study performed in this chapter has been published in "Jour. of Inst. of Maths. & Computer Sciences", (2008).

The third chapter presents mathematical system representing two similar parallel units in cold standby system with four modes of each unit normal (N) total hardware failures (F) and human errors (H) and additional partial hardware failure (PF) .The mean-time to system failure, reliability to system and steady-state availability are obtained.

Using the transitions of the Markov process to the up states and then the Laplace transform of the various state probabilities have been derived also the reliability is obtained by the inversion process. The failure times of operating/spare units and repair time of failed units are exponential distributed .Computer programs are used to calculate mean-time to system failure, reliability to system and steady-state availability. Also the effects of additional partial hardware failure on the system performance are shown graphically. The study performed in this chapter has been published in "Jour. of Inst. of Maths. & Computer Sciences" (2008).

The fourth chapter considers a repairable complex system consisting of two subsystems A and B. The subsystem A has a two-unit active parallel system whereas the subsystem B has one unit alone. The two subsystems are arranged in series. Both the units of subsystem A suffer two types of failure viz; hardware and human whereas subsystem B suffers only one type of failure. The Laplace transforms of the various state probabilities have been derived and then reliability is obtained by the inversion process. Moreover, an important parameter of reliability, i.e. MTTF (mean time to failure), system availability and steady-state availability are derived. The failure times of operating units and repair time of failed units are exponential distributed. The effects of additional repair in this system performance are shown in tables and graphically. The study performed in this chapter has been accepted for publication in "Journal of Mathematics and Statistics" (2009).

The fifth chapter discusses a system for two-dissimilar parallel units cold standby redundant system with four different modes [normal, activation, partial failure and total failure]. System fails when both units

fail totally. When the operative unit fails partially or after random time during the activation time the use of the partially failed unit is made. As soon as the cold standby unit becomes active, start operation and the partially failed unit stops operation and goes for repair. Using the evaluate Markov-renewal process we the following reliability characteristics of interest to system designers as well as operations managers. The plats for the system reliability, system mean time to failure and system availability and drawn for various parametric values for same special cases. Case 1 assumes that the activating time and repair times are exponential distributed. Case 2 assumes that the two units are similar with exponential distributions. The results obtained by Gole and Gupta (1983) are derived from the present chapter as particular cases.

The sixth chapter investigates a two-dissimilar-unit warm standby system where each unit works in two different modes normal and total failure. The failure, the repair of units and the repair of a switch time distributions are assumed to different arbitrary distributed, the probability that the switch works at the time of need is q = (1-p). Repair of a total failure unit, failed from operative or standby state continues when the other unit enters the total failure mode, when an operative unit fails, switch is used to disconnect the failed unit and connect the standby unit if it is operative. A single repair facility is available, priority for repair being given to transfer switch. After repair of a unit or the switch works like a new one, repair time distribution of a unit failed from the standby state is different from that of the unit failed from operative state. Switch failure occurs in a non-regenerative state, for the first time. Using the semi-Markov process technique and the results of the regenerative process, various measures of the system effectives as mean time to

system failure, pointwise availability, steady state availability and busy period analysis are found out. The results by Mokaddis (1990) are derived from the present results as a special case.

This seventh chapter deals with the reliability analysis two mathematical models representing electric power systems operating in fluctuating outdoor weather (i.e., normal and stormy weather) and compared between two models. Model I deals the reliability analysis of a single-server two-unit cold standby, Model II deals the reliability analysis of a single-server two-unit warm standby .for two systems with two different modes [normal, total failure]. The two models occur when both the units fail totally. The failure rate and failed repair rate of a unit are constants. Laplace transforms of the various state probabilities have been derived and then reliability is obtained by the inversion process. Moreover, an important parameter of reliability, i.e. MTTF (mean time to failure), availability and steady-state availability are derived. The failure times of operating/spare units and repair time of failed units are exponential distributed. Certain important results have compared between two models, we note that the model I (with cold standby) is better the model II (with warm standby). The study performed in this chapter has been accepted for publication in "Journal of Mathematics and Statistics" (2010).

# ON PROBABILISTIC ANALYSIS OF DIFFERENT REDUNDANT SYSTEMS AND THEIR APPLICATIONS

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#### CHAPTER ONE

# DEFINITIONS AND SOME CONCEPTS FOR THE RELIABILITY THEORY

#### 1.1 INTRODUCTION

Reliability engineering is a vast field and it has grown significantly during the past five decades (since World War II).In the pursuit of maximum efficiency in the operation of large-scale military and commercial engineering systems, the complexity of equipments and configurations has been an ever increasing phenomenon. Reliability theory's composed of the systematic procedures based on analytical techniques employed to ensure the operational efficiency of such complex system. As a quantitative measure, reliability is defined as the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions. Redundancy, renewal and maintenance are considered in order to maximize the efficiency of the operation of systems. The mean lifetime and the availability coefficient are considered as quantitative measures for the efficiency of systems. Application of reliability techniques is gaining importance in all branches of engineering because of its effectiveness in the detection, prevention, and correction of failures in the design, manufacturing, and operational.

This chapter presents some definitions, basic characteristics and some concepts for the reliability theory. the distribution of a continuous nonnegative random variable T are used extensively in the reliability literature; the reliability function, the hazard rate, the

cumulative hazard function and the mean residual life function, combinatorial aspects are defined. The transforms, system of linear first orders differential equations, standby redundant system and the Markov models are defined. Also we taking some examples and graphs to showing the basic characteristics.

#### 1.2 BASIC CONCEPTS OF RELIABILITY THEORY

Just as in other branches of science, the basic concepts of reliability theory are understood by describing the relationships among them. By a unit is meant an element, a system, a part of a system or the like. The operation of a unit means the set of all phases of its existence: transportation, maintenance, preparation for a specified use, servicing and repair. The concept of the reliability of a unit is connected in very real way with the concept of its quality. The quality of a unit is the set of properties defining the degree of suitability of the unit for a specified use. Thus, the concept of the quality of a unit depends in a very real way on the manner in which it is used. By the reliability of a unit we mean the ability of the unit to maintain its quality under specified conditions of use. In other words, reliability is a property which is extended in time. Reliability is determined by quality and operating conditions. One of the basic concepts in reliability theory is that of failure and failure-free operation. Failure-free operation is the ability of the unit to keep its ability to function (i.e. not to have failures) throughout a specified period of time under specified conditions. A failure is the partial or total loss or modification of those properties of the units in