



Cairo University

EFFECT OF SELF ASSEMBLED QUANTUM DOTS ON
CARRIER MOBILITY, WITH APPLICATION TO
MODELING THE DARK CURRENT IN QUANTUM DOT
INFRARED PHOTODETECTORS

By

Sarah Youssef Abdelrahman Ahmed Abdelrahman

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
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MASTER OF SCIENCE
in
Engineering Physics

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
GIZA, EGYPT
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Title of Thesis:

**Effect of Self Assembled Quantum Dots on Carrier Mobility, with
Application to Modeling the Dark Current in Quantum Dot Infrared
Photodetectors**

Key Words: Quantum dot infrared photodetectors, quantum dots, mobility, dark current, Boltzmann transport equation

Summary:

A theoretical method for calculating the electron mobility in quantum dot infrared photodetectors is developed. The mobility calculation is based on a time-dependent, finite-difference solution of the Boltzmann transport equation in a bulk semiconductor material with randomly positioned conical quantum dots. The quantum dots act as scatterers of current carriers, resulting in limiting their mobility. The calculated values of the mobility are used in a recently developed generalized drift-diffusion model for the dark current of the device in order to fix the overall current scale. The results of the model are verified by comparing the predicted dark current characteristics to those experimentally measured and reported for actual InAs/GaAs quantum dot infrared photodetectors. Finally, the effect of the several relevant device parameters, including the operating temperature and the quantum dot average density, is studied.

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Dedication

This thesis is dedicated to my parents, sisters, husband, and my beloved daughter.

Table of Contents

TABLE OF CONTENTS.....	I
LIST OF TABLES	V
LIST OF FIGURES	VI
LIST OF SYMBOLS AND ABBREVIATIONS.....	VII
ABSTRACT	X
CHAPTER 1 : INTRODUCTION AND OVERVIEW.....	1
1.1. INFRARED SYSTEMS	2
1.1.1. Infrared systems applications	3
1.2. PHOTODETECTORS	4
1.2.1. Infrared photodetectors.....	5
1.2.1.1. Intrinsic Interband Photodetectors.....	5
1.2.1.2. Extrinsic photodetectors.....	6
1.2.1.3. Intersubband photodetectors.....	6
1.3. QUANTUM DOTS (QDs)	7
1.3.1. Fabrication of QD.....	8
1.3.1.1. Etching	8
1.3.1.2. Semiconductor microcrystal.....	8
1.3.1.3. Selective growth.....	8
1.3.1.4. Self-organized growth (Stranski-Krastanov) ^[45]	9
1.4. MODELING OF THE DARK CURRENT OF QDIPS.....	9
1.4.1. Previous work of dark current modeling in QDIP.....	9
1.4.2. The carrier mobility.....	11
1.4.2.1. Boltzmann's transport equation (BTE).....	11
1.4.2.2. Methods of solving Boltzmann's transport equation.....	12
1.4.2.2.1. Relaxation time approximation	12
1.4.2.2.2. Spherical Harmonic method.....	12
1.4.2.2.3. Scattering matrix approach.....	12
1.4.2.2.4. Cellular Automata.....	12
1.5. THE THESIS PURPOSE.....	13
1.6. CONCLUSIONS	13
CHAPTER 2 : THE COMPLETE MODEL OF THE DARK CURRENT AND MOBILITY OF QDIPS	14
2.1. THE QDIP STRUCTURE.....	14
2.2. THE DARK CURRENT MODEL FOR QDIPS.....	15
2.3. ELECTRON MOBILITY CALCULATION FOR QDIP	19
2.3.1. Relaxation time approximation	22
2.3.2. Time-domain finite-difference method	25
2.3.3. The matrix element of perturbation by conical QDs.....	30
2.3.4. The effects of other Scattering mechanisms.....	33
2.4. CONCLUSION	35
CHAPTER 3 : RESULTS AND DISCUSSIONS	36
3.1. THE NUMERICAL CALCULATIONS OF THE CARRIER MOBILITY	36
3.1.1. The matrix element.....	38
3.2. CARRIER MOBILITY FOR SPHERICAL QDS	39
3.2.1. The distribution function.....	39
3.2.2. Velocity and mobility	41

3.3.	CARRIER MOBILITY FOR CONICAL QD.....	43
3.3.1.	The distribution function.....	43
3.3.2.	Velocity and mobility.....	44
3.3.2.1.	The effects of different scattering mechanisms on electron mobility	46
3.3.3.	Effects of QD density and dimensions on the mobility of Conical QD Structure.....	47
3.4.	RESULTS OF DARK CURRENT.....	49
3.4.1.	The comparison with the experimental results of dark current	49
3.4.2.	Effects of QD density on the dark current.....	50
3.4.3.	Effects of temperature on the dark current	50
3.5.	CONCLUSIONS	51
SUMMARY AND CONCLUSIONS		53
I.	THE THEORETICAL METHOD FOR CALCULATING THE ELECTRON MOBILITY IN QDIPS	53
II.	THE COMPLETED MODEL OF THE DARK CURRENT.....	53
APPENDIX A: MOBILITY CALCULATIONS IN THE PRESENCE OF SCATTERING MECHANISMS OTHER THAN QUANTUM DOTS.....		55
I.	THE SCATTERING TERM OF THE LONGITUDINAL OPTICAL PHONONS	55
II.	EMPIRICAL EXPRESSIONS OF MOBILITY LIMITED BY DIFFERENT SCATTERING MECHANISMS ^{[79], [80]}	57
•	Mobility limited by doping (impurities).....	57
•	Mobility limited by optical phonons (for GaAs).....	58
•	Mobility limited by acoustic phonons (for GaAs).....	58
REFERENCES.....		59

List of Tables

TABLE 3-1: THE MOBILITY OF CONICAL AND SPHERICAL QDS UNDER THE SAME CONDITIONS	45
TABLE 3-2: SCATTERING RATES DUE TO EACH SCATTERING MECHANISM	46
TABLE A- 1: PARAMETERS OF THE MOBILITY EXPERIMENTAL MODEL (FOR GAAS) ^{[78], [79]}	58

List of Figures

FIGURE 1-1: BLACKBODY RADIATION FOR DIFFERENT TEMPERATURES	3
FIGURE 1-2: A PHOTODETECTOR RECEIVING INCIDENT PHOTONS AND CONVERTING THEM INTO AN ELECTRICAL SIGNAL.....	4
FIGURE 1-3: HGCDTE ENERGY BAND DIAGRAM SHOWING ITS NARROW BAND GAP	5
FIGURE 1-4: A BAND DIAGRAM SHOWING THE QUANTUM WELL STRUCTURE.....	7
FIGURE 1-5: A BAND DIAGRAM SHOWING THE QUANTUM DOT STRUCTURE.....	7
FIGURE 1-6: BAND STRUCTURE OF A SINGLE QUANTUM DOT STRUCTURE.....	7
FIGURE 1-7: SELF ORGANIZED DOTS FORMED USING STRANSKI-KRASTANOV GROWTH METHOD.	9
FIGURE 2-1: THE SCHEMATIC OF THE QDIP STRUCTURE.....	14
FIGURE 2-2: BASIC CELL OF CONICAL QD WITH RADIUS r_b AND HEIGHT h_b , CONTAINING A QD WITH RADIUS r_d AND A HEIGHT OF h_d	18
FIGURE 2-3: BAND STRUCTURE SHOWING THE CONDUCTION BAND OFFSET BETWEEN THE BARRIER AND QUANTUM DOT MATERIALS.....	21
FIGURE 2-4: THE SCATTERING PROCESS DUE TO SPHERICAL SCATTERER.....	24
FIGURE 2-5: THE RELATION BETWEEN k , k' , AND q IN THE CASE OF ELASTIC COLLISION (LIKE QD SCATTERING) IN THE K-SPACE.....	29
FIGURE 2-6: A SPHERICAL QD WITH RADIUS r_o	29
FIGURE 2-7: A CONICAL QD WITH BASE RADIUS r_d , HEIGHT h_d , AND HALF APEX ANGLE θ_o	31
FIGURE 3-1: THE FLOW CHART DESCRIBING THE COMPLETE METHOD TO DETERMINE THE VELOCITY-ELECTRIC FIELD RELATION.	37
FIGURE 3-2: THE MATRIX-ELEMENT AMPLITUDE SQUARED $V_{k',k}^2$ OF SPHERICAL AND CONICAL QDS OF THE SAME VOLUME FOR DIFFERENT SCATTERING ANGLES AND DIFFERENT $\theta_k, \phi_{k'}$	38
FIGURE 3-3: THE DISTRIBUTION FUNCTION $f(k)$ VERSUS THE MAGNITUDE OF K-VECTOR AT ELECTRIC FIELD = 9×10^6 V/M AND $\theta_k = 18^\circ$	39
FIGURE 3-4: THE DISTRIBUTION FUNCTION VERSUS θ_k AT ELECTRIC FIELD = 9×10^6 V/M AND $k = 10^8$ M ⁻¹	40
FIGURE 3-5: THE DISTRIBUTION FUNCTION VERSUS THE MAGNITUDE OF K-VECTOR AT ELECTRIC FIELD = 10^4 V/M AND $\theta_k = 18^\circ$	40
FIGURE 3-6: THE DISTRIBUTION FUNCTION VERSUS THETA (θ_k) AT ELECTRIC FIELD = 10^4 V/M AND $k = 10^8$ M ⁻¹	41
FIGURE 3-7: THE VELOCITY AS A FUNCTION OF TIME AT DIFFERENT VALUES OF THE APPLIED ELECTRIC FIELD	42
FIGURE 3-8: THE VELOCITY-ELECTRIC FIELD RELATION FOR DIFFERENT NUMBERS OF TIME STEPS.....	42
FIGURE 3-9: THE DISTRIBUTION FUNCTION VERSUS THE ELECTRON ENERGY FOR $\theta_k = 36$ AND ELECTRIC FIELD $\xi = 6 \times 10^5$ V/M AT DIFFERENT TIME STEPS.	43
FIGURE 3-10: THE DISTRIBUTION FUNCTION VERSUS WAVE VECTOR ANGLE (θ_k) FOR ELECTRON ENERGY $\varepsilon_k = 1.4 \times 10^8$ J AND ELECTRIC FIELD $\xi = 6 \times 10^5$ V/M AT DIFFERENT TIME STEPS.....	44
FIGURE 3-11: THE VELOCITY OF THE CARRIER FOR DIFFERENT VALUES OF THE APPLIED ELECTRIC FIELD.	45
FIGURE 3-12: THE CALCULATED TEMPERATURE FOR QD SCATTERING ONLY	47
FIGURE 3-13: CALCULATED CARRIER MOBILITY IN QDIP VERSUS THE QD BASE RADIUS FOR DIFFERENT QUANTUM DOT DENSITIES.	48
FIGURE 3-14: CALCULATED CARRIER MOBILITY IN QDIP VERSUS THE QD HEIGHT FOR DIFFERENT QUANTUM DOT DENSITIES.	48
FIGURE 3-15: CALCULATED DARK CURRENT AND EXPERIMENTAL DATA IN ^[10] OF QDIP.....	49
FIGURE 3-16: THE DARK CURRENT IN QDIP VERSUS BIAS VOLTAGE FOR DIFFERENT VALUES OF QUANTUM DOT DENSITY AT T=77 K.....	50
FIGURE 3-17: THE DARK CURRENT IN QDIP VERSUS BIAS VOLTAGE FOR DIFFERENT OPERATING TEMPERATURES (QD DENSITY= 2.5×10^{22} M ⁻³ , $r_d = 10$ NM, AND $h_d = 7$ NM).	51
FIGURE A-1: DIFFERENT CASES OF SCATTERING	56

List of Symbols and Abbreviations

QDIP	Quantum dot infrared photodetector
QWIP	Quantum well infrared photodetector
BTE	Boltzmann's transport equation
QD	Quantum dot
QW	Quantum well
LED	Light emitting diode
HgCdTe	Mercury Cadmium Telluride
PbSe	Lead selenide
PbTe	Lead telluride
PbS	Lead sulfide
GaAs	Gallium arsenide
InAs	Indium arsenide
InGaAs	Indium gallium arsenide
CdSe	Cadmium selenide
CdS	Cadmium sulfide
CuBr	Copper bromide
CuCl	Copper chloride
ε_g	Bandgap energy
J	Electric current density
e	Electron charge
μ	Electron mobility
n	Electron concentration

ξ	Electric field
D_n	Electron diffusion coefficient
N_c	Effective density of states in conduction band
$F_{1/2}$	Fermi Dirac integral of order half
ε_f	Fermi energy level
ε_c	Conduction band edge
K_B	Boltzmann's constant
T	Temperature
V	Electrostatic potential
Γ	Gamma function
ρ	Volumetric charge density
ϵ	Barrier material permittivity
N_D^+	Ionized donor concentration
N_D	Donors concentration
n_{QD}	Quantum dot density
$\langle N \rangle$	Average dot filling
f	Electron distribution function
\vec{k}	Wave vector

\vec{r}	Real space vector
\hbar	Reduced Planck's constant
$W_{k',k}$	Transition rate from one state to another
$V_{k',k}$	Matrix element of perturbation
V_b	Conduction band offset
τ_m	Momentum relaxation time
τ_i	Single- particle life time
α	Scattering angle
Δt	Time discretization step
$\langle v \rangle$	Drift velocity
r_b	Basic cell radius
h_b	Basic cell height
r_d	Quantum dot base radius
h_d	Quantum dot height
θ_o	Half apex angle of quantum dot
m^*	Effective mass in the barrier material

Abstract

The quantum dot infrared photodetector (QDIP) is one of the most promising candidates for infrared photodetection. According to theoretical studies, its performance is expected to be superior to that of an infrared photodetector making use of more mature technologies such as an HgCdTe or a quantum well infrared photodetector (QWIP).

Although there is a lot of experimental and theoretical research work on QDIPs, there is no complete theoretical model for the device, even one for its dark current which is perceived as noise and limits its sensitivity and maximum operating temperature. In a previous work, an almost microscopic generalized drift-diffusion model of the dark current characteristics has been developed, and shown to successfully predict the dark current dependence on the biasing voltage up to an overall scale factor that could not be determined due to the lack of reliable values of the carrier mobility in the presence of the quantum dots.

The purpose of this thesis is to develop a theoretical method for calculating the electron mobility in QDIPs and use it to complete the above-mentioned dark current model. The mobility calculation is based on a time-dependent, finite-difference solution of the Boltzmann transport equation (BTE) in a bulk semiconductor material with randomly-positioned conical quantum dots. The quantum dots act as scatterers of current carriers (conduction-band electrons in our case), resulting in limiting their mobility. In fact, carrier scattering by quantum dots is typically the dominant factor in determining the mobility in the active region of the quantum dot device. After studying the dependence of the mobility on various relevant parameters, the completed model is verified by comparing the dark current characteristics it predicts to those experimentally measured and reported for actual InAs/GaAs devices. Finally, the effect of the various relevant device parameters, including the operating temperature and the quantum dot average density, is studied. The next step should be the extension of the model to the calculation of the photocurrent.

Keywords: Quantum dot infrared photodetectors, quantum dots, mobility, dark current, Boltzmann transport equation

Chapter 1 : Introduction and overview

The nanostructures have been developed decades ago and they have been expected to significantly improve the existing devices as they have a lot of promising properties, especially the quantum dots^{[1]-[6]} which are expected to be superior to competing nanostructures. In Quantum dot-structures, the size of quantum dot is comparable to molecular size and the band gap can be accurately tuned by changing the shape and size of QDs and the composition of the material. The tunability of the band gap is a very important optoelectronic property as it allows the tuning of the absorption and emission spectrum. Thus, the quantum dots have a lot of applications in optoelectronics, and one of the most important of them is quantum dot infrared photodetector.

The quantum dot infrared photodetector (QDIP) is one of the most promising candidate for infrared photodetection, as it is expected to have a superior performance to more mature infrared photodetectors such as mercury cadmium telluride photodetectors (HgCdTe) and quantum well infrared photodetectors (QWIP)^[7]. The QDIP is expected to have lower dark current which is considered as noise that limits the sensitivity of the photodetector, in addition, it can operate at higher temperatures^[7], and can detect the normal incident radiation while other types like QWIP cannot^[8].

Modeling of QDIP is very important to optimize its performance. One of the most important and significant parameters to be modeled is the dark current. The dark current of photodetectors is perceived as noise since it is the current that is generated and flows in the photodetector in the absence of incident optical signal. In addition, dark current is considered as the main factor limiting the maximum operating temperature of the photodetector.

There are many experimental^{[9]-[15]} and theoretical^{[16]-[31]} studies of QDIPs which mainly aim to develop models of important parameters such as the dark current to predictively analysis the performance of QDIPs. Most of those models proposed for the device consider the device dark current to be space-charge limited, and are generally semi-phenomenological in the sense that some of their parameters are not related to the structure and the material properties of the device but are fitted to experimental results^{[16]-[27]}. The few suggested fully microscopic models deal with either a highly reduced geometry^[28] or more complicated structures designed to reduce the dark current^{[29]-[31]}.

In^[32], a model for calculating the dot filing and the dark current profile of QDIPs has been presented. This model is almost microscopic as it gives the correct dependence of the dark current on the applied biasing apart from the overall current scale which could not be fixed in the lack of reliable values of the carrier mobility in the presence of the conical QDs; and this was the main motivation to develop a complete theoretical method for the calculation of the mobility for QDIPs. In addition, calculating the mobility is so necessary to determine other important parameters such as the electrical conductivity, diffusion constant, drift velocity, and the current flows through a device. Moreover, the mobility itself is an important parameter for any optoelectronic device as it is the measure of how fast carriers flow in the device due to an electric field. It is the key parameter in transport simulation and need to be determined in any electrical device.

There is no study to determine mobility or other related parameters such as velocity in QDIPs and it is usually determined empirically; however, there are some theoretical studies^{[33],[34]} done to determine the mobility or mobility-related parameters such as velocity or relaxation time in other devices and structures. Unfortunately, most of these

studies use approximated methods that might not be valid for many realistic devices or scattering mechanisms.

As there's no complete theoretical model of dark current and no reliable values of electron mobility in QDIPs, we were motivated to present in this thesis a fully theoretical procedure for calculation the electron mobility in quantum dot infrared photodetectors, and use it to complete the dark current model presented in the previous work ^[32] to get the values of dark current, not only its profile. The completed dark current model and the mobility calculations are generic and can be applied for other QDIPs with different semiconductor compounds, number of quantum dot layers, quantum dot density in layers, or dimensions and with some modifications it can be used with other shapes of quantum dots such as semi-spheroid or pyramidal quantum dots or even with other optoelectronic devices based on quantum dots. In addition, the mobility calculations can be extended to calculate important parameters such as the photocurrent of the device which is the current flowing through the photodetector due to exposure to illumination.

After this brief introduction of our study, we will have an overview over quantum dot infrared photodetectors. First, we will discuss the infrared systems and their applications, infrared photodetectors, quantum dots, their applications and their fabrication techniques. Then, we represent a survey over modeling of QDIP, especially modeling of its dark current. After that, we will discuss the mobility and the Boltzmann's transport equation as it is the core of the method of electron mobility calculation for quantum dot devices that will be developed in chapter 2.

1.1. Infrared systems

The electromagnetic spectrum is divided into regions according to their wavelengths; the ultraviolet, the visible light, and the infrared. Infrared radiation is the region of electromagnetic spectrum with wavelengths ranging from 700nm to 1mm. As discovered experimentally, the different objects emit different radiations depending on the temperature of these objects. The concept of blackbody (like sun or human body) radiation gives an explanation of this phenomena as shown in Figure 1-1 where the intensity of the blackbody radiation is plotted for different temperatures. It shows that most of blackbodies have their radiation peaks in the infrared region (700 nm to 1 mm) except at very high temperatures.