بسم الله الرحمن الرحيم

شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

﴿ قـــسـم ﴾

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Immersions into special types of Riemannian manifolds

A Thesis in Mathematics
Submitted for the
Degree of Doctor of Philosophy
(Pure Mathematics)

By

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Preface

Levi-Civita has proved that given a manifold M with a Riemannian metric g there is a unique connection ∇ satisfying the following two properties:

- $\nabla g = 0$, i.e., the connection is metric.
- For $X, Y \in \chi(M)$, $T(X,Y) = 0$, i.e., the connection is symmetric.

The connection is then called a Riemannian connection.

Friedmann (1924) and Schouten (1954) [**19**] introduced the idea of semi-symmetric linear connection on a differentiable manifold. Hayden (1932) [**11**] introduced semi-symmetric metric connection on a Riemannian manifold and this was further developed by Yano (1970) [**20**], Imaii (1972) [**13**], Nakao (1976) [**16**], Amur and Pujar (1978) [**4**].

In 1992, Agashe and Chafle [**2**] defined a semi-symmetric non-metric connection $\overset{*}{\nabla}$ on a Riemannian manifold M and defined the curvature tensor of M with respect to this semi-symmetric non-metric connection. They obtained a relation connecting the curvature tensors of M with respect to semi-symmetric non-metric connection and the Riemannian connection. Further, they obtained

the first and second Bianchi identities associated with the semi-symmetric non-metric connection.

In this thesis we continue the study of this connection $\overset{*}{\nabla}$ together with the semi-symmetric metric connection $\tilde{\nabla}$. We also study the main geometric properties of submanifolds of Riemannian spaces endowed with non-Riemannian connections. The thesis consists of five chapters, an introduction together with four chapters which contain the main results we have established.

In the introduction, we give the fundamental concepts and definitions which we deal with throughout the thesis. The notions of manifolds, submanifolds, tangent vector fields, tensors, immersion, general linear connection and Riemannian connection are all covered in the sections of this chapter.

In Chapter 2, we give a relation between the Christoffel symbols of the Riemannian connection ∇ and the connection coefficients of $\overset{*}{\nabla}$. We also give relations between Christoffel symbols of ∇ and that of the semi-symmetric metric connection $\tilde{\nabla}$. We study Jacobi fields and Lie derivatives with respect to $\overset{*}{\nabla}$. We end the chapter with a section on sectional curvature and show the difficulty in having a well-defined function that plays the role of sectional curvature in the case of semi-symmetric non-metric connection $\overset{*}{\nabla}$.

In Chapter 3, we consider a submanifold of a Riemannian manifold with a semi-symmetric non-metric connection $\overset{*}{\nabla}$. We study

the geometry of the submanifold with respect to the geometry of the ambient space. The results of this chapter have been given as a talk in the "Third Mathematical conference on Geometry, Topology and Applications", Cairo University, 20-24 December 1997. It has also been accepted for publication in the Bulletin of the Faculty of Science, Ain Shams University, 1998.

In Chapter 4, we consider the product of two manifolds each endowed with a semi-symmetric non-metric connection as well as the product of two manifolds each with a semi-symmetric metric connection. We give the curvature and torsion tensors in this case in relation with the Riemannian case. The main results of this chapter have been submitted for publication in the Kyungpook Mathematical journal (Korea).

In Chapter 5, we are interested in the problem of a deformation of a product manifold. The initial problem was investigated in [9] for the case of a single manifold with Riemannian connection. We were able to give better results due to the idea of taking products of manifolds. We suggest studying this problem in the case of semi-symmetric non-metric connection adopting the results of chapter four. The main results of this chapter have been accepted for publication in "Communications de la Faculté des Sciences de l'Université d'Ankara", Series A1, Mathematics and Statistics, Vol 46(1997) [5].

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