



# **ELECTROMAGNETIC WAVE-PARTICLE INTERACTION USING CAVITY MODAL EXPANSION**

By

Ahmed Farghaly Abdelrahman Abdelshafy

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfilment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
Electronics and Communications Engineering

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**Title of Thesis:**

ELECTROMAGNETIC WAVE-PARTICLE INTERACTION  
USING CAVITY MODAL EXPANSION

**Key Words:**

Wave-Particle Interaction; Cavity Modal Expansion(CME); Particle in Mode(PIM); Cherenkov radiation

**Summary:**

A new technique to simulate the interaction between the electromagnetic fields and charged particles. This technique is based on Cavity Modal Expansion whereby the known solenoidal and irrotational eigenmodes of a canonical cavity are employed to solve different electromagnetic problems. In this study, the technique is employed to solve a cold electromagnetic problem like circular waveguide excited by an axial current probe to obtain the transient solution in order to validate time domain model. Also, it is employed to solve a hot electromagnetic problem, where the interaction between the fields and particles occurs, as Cherenkov radiation to obtain the dynamic behavior of the particle as well as the electromagnetic fields radiation.





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Ahmed.



# Dedication

*This thesis dedicated to my family and friends.*



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