



Ain Shams University,  
Faculty of Engineering,  
Irrigation and Hydraulics Department.

**Modeling Seepage Effects In Heterogeneous Soil Under  
Heading-Up Structures Using An Experimental And  
Numerical Methodology**

Thesis  
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**By**  
**Eng. Doaa Anas El-Molla**  
B.sc. Civil Engineering, Ain Shams University (2004)  
M.sc. Civil Engineering, Ain Shams University (2009)

**Supervised By**  
**Prof. Dr. Nahla Mohamed AboulAtta**  
Professor & Head of the irrigation and Hydraulics Dept.  
Faculty of Engineering, Ain Shams University

**Dr. Ghada Mahmoud Samy**  
Associate Professor, Irrigation and Hydraulics Department  
Faculty of Engineering, Ain Shams University

**Dr. Mohamed Abdel-Hamid Gad**  
Associate Professor, Irrigation and Hydraulics Department  
Faculty of Engineering, Ain Shams University

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## **Statement**

This thesis is submitted to the Irrigation and Hydraulics Department, Faculty of Engineering, Ain Shams University in partial fulfillment of the Ph.D. degree in Civil Engineering.

The work in this thesis was carried out in the Irrigation and Hydraulics Department, Faculty of Engineering, Ain Shams University.

No part of this thesis has been submitted for a degree or a qualification at any other university or institution.

Date:

Name: Doaa Anas Mohamed El-Molla

Signature:

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## **Abstract**

Seepage of water under the aprons of heading-up structures is one of the most important factors that should be carefully studied while designing any heading-up structure. Seepage can cause different problems related to uplift pressure and undermining and piping. These problems threaten the stability of the structures and can lead to their failure. In nature, soil is normally heterogeneous rather than uniform. One of the special cases of soil heterogeneity is stratified soil where soil layers are usually horizontal as most soils deposit in this manner.

This study introduces a new electric analogue experimental modeling methodology to simulate the effect of soil stratification under the apron of heading-up structures on the safety against uplift and piping. This is achieved through making different scenarios for the hydraulic conductivity values of the layers and their configurations under the apron. Both SEEP2D and electric analogue models are applied to study seepage under heading-up structures occurring on both a single layer and stratified soils. The two models are compared to check the accuracy of the electric analogue in simulating seepage in a trial to add more verification cases of the electric approach.

The results showed that the pressure distribution under the floor of heading up structures is actually non-linear, and accordingly, the linear assumption can be a weak assumption to study seepage under large heading up structures and dams. Stratification is found to affect the head distribution under the apron, and the total flow rate.

The results of the electric analogue are very promising and of good accuracy compared to numerical modeling. This gives rise to the applicability of the electric analogue to study seepage under heading up

structure. This can become very useful if future studies could reveal relations between the electric analogue model and seepage failures.

This research also includes a comparison between 2D and 3D numerical models, in an attempt to evaluate the effect of neglecting the third dimension in studying seepage under an apron for both laterally homogeneous and heterogeneous conditions. Charts relating the exit gradient resulting from 2D model to that from the 3D model at centerline and sides of the apron's width for different values of  $H/B$  and different structure configurations including the presence and lack of an upstream sheet pile have been developed. In addition, the effects of downstream wing walls (guide walls) existence on seepage are also assessed.

The results prove that studying seepage in 3D can significantly become critical over the traditional 2D approach. This makes 3D simulation essential when studying seepage under large heading up structures (e.g. large dams) especially in complicated 3D configurations and lateral heterogeneity and/or anisotropy cases.

In this research piping problem under hydraulic structures is also reconsidered and a modification to Ojha (2003) critical velocity based piping model is suggested in an attempt to improve the accuracy of the critical hydraulic gradient estimation.

# **Table of Contents**

|                                                                |             |
|----------------------------------------------------------------|-------------|
| <b>Statement .....</b>                                         | <b>I</b>    |
| <b>Acknowledgement.....</b>                                    | <b>II</b>   |
| <b>Abstract.....</b>                                           | <b>III</b>  |
| <b>Table of Contents .....</b>                                 | <b>V</b>    |
| <b>List of Figures .....</b>                                   | <b>VII</b>  |
| <b>List of Tables.....</b>                                     | <b>XIII</b> |
| <b>Notations .....</b>                                         | <b>XIV</b>  |
| <b>Chapter (1): Introduction .....</b>                         | <b>1</b>    |
| 1.1 GENERAL.....                                               | 1           |
| 1.2 OBJECTIVE OF THE STUDY .....                               | 2           |
| 1.3 THESIS OUTLINE .....                                       | 3           |
| <b>Chapter (2): Literature Review.....</b>                     | <b>4</b>    |
| 2.1 SEEPAGE PROBLEM.....                                       | 4           |
| 2.1.1 Uplift .....                                             | 4           |
| 2.1.2 Undermining .....                                        | 5           |
| 2.1.3 Piping .....                                             | 5           |
| 2.1.4 Erosion of the bed .....                                 | 6           |
| 2.2 CRITICAL GRADIENT EQUATIONS .....                          | 7           |
| 2.2.1 Theoretical background of critical gradient models ..... | 7           |
| 2.2.2 Critical head using Darcy's law.....                     | 14          |
| 2.2.3 Dependence of critical head on porosity .....            | 15          |
| 2.3 METHODS OF SOLUTION FOR CONFINED FLOW PROBLEMS .....       | 15          |
| 2.3.1 Exact methods .....                                      | 15          |
| 2.3.2 Approximate methods .....                                | 16          |
| 2.3.3 Experimental methods (Solution by Analogies).....        | 18          |
| 2.3.4 Analytical methods.....                                  | 22          |
| 2.3.5 Numerical methods .....                                  | 23          |
| 2.4 SEEP2D.....                                                | 26          |
| 2.4.1 Governing equation of SEEP2D .....                       | 27          |
| 2.4.2 Displaying Seep2D results.....                           | 28          |
| 2.5 SEEP3D .....                                               | 28          |
| 2.5.1 Flow law used by Seep3D .....                            | 29          |
| 2.5.2 Governing equations.....                                 | 30          |

|                                                                                                          |            |
|----------------------------------------------------------------------------------------------------------|------------|
| 2.5.3 Finite element equations.....                                                                      | 30         |
| 2.6 SUMMARY OF THE PREVIOUS RESEARCH WORK.....                                                           | 32         |
| 2.7 UNIQUENESS OF THE PRESENT STUDY .....                                                                | 44         |
| <b>Chapter (3): Comparative Analysis Between Electric Analogue and a 2D Numerical Seepage Model.....</b> | <b>45</b>  |
| 3.1 INTRODUCTION.....                                                                                    | 45         |
| 3.2 DESCRIPTION OF THE MODELS .....                                                                      | 46         |
| 3.2.1 SEEP2D .....                                                                                       | 46         |
| 3.2.2 Electric Analogue.....                                                                             | 47         |
| 3.3 APPLICATION OF THE MODEL.....                                                                        | 50         |
| 3.3.1 For one soil layer under the apron .....                                                           | 50         |
| 3.3.2 For two soil layers under the apron.....                                                           | 51         |
| <b>Chapter (4): Comparative Analysis Between 2D And 3D Numerical Seepage Models.....</b>                 | <b>81</b>  |
| 4.1 INTRODUCTION.....                                                                                    | 81         |
| 4.2 THEORETICAL BACKGROUND .....                                                                         | 82         |
| 4.3 MODELS APPLICATION .....                                                                             | 83         |
| 4.3.1 3D Model Verification .....                                                                        | 84         |
| 4.3.2 Effect of bed width .....                                                                          | 85         |
| 4.3.3 Effect of lateral heterogeneity of the 3D domain .....                                             | 90         |
| 4.3.4 Effect of downstream wing walls.....                                                               | 93         |
| <b>Chapter (5): Piping Determination Using the Critical Velocity Concept .....</b>                       | <b>122</b> |
| 5.1 INTRODUCTION.....                                                                                    | 122        |
| 5.2 MATHEMATICAL DERIVATION .....                                                                        | 123        |
| 5.3 TEST CASES.....                                                                                      | 126        |
| 5.4 CLOSING REMARKS .....                                                                                | 129        |
| <b>Chapter (6): Conclusions and Recommendations.....</b>                                                 | <b>132</b> |
| 6.1 CONCLUSIONS .....                                                                                    | 132        |
| 6.2 RECOMMENDATIONS .....                                                                                | 135        |
| <b>References .....</b>                                                                                  | <b>136</b> |
| APPENDIX (A): STRATIFIED ELECTRIC ANALOGUE EXPERIMENTAL READINGS. ....                                   | 142        |
| APPENDIX (B): TABLES COMPARING HEAD VALUES OBTAINED FROM ELECTROLYTIC TANK AND SEEP2D .....              | 147        |
| APPENDIX (C): SEEP3D EXIT GRADIENT RESULTS ACROSS THE WIDTH OF THE APRON.....                            | 151        |

## **List of Figures**

|                                                                                                                                                |    |
|------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure (2-1): Definition sketch for sand boil formation.....                                                                                   | 9  |
| Figure (2-2): The Flow Net.....                                                                                                                | 17 |
| Figure (2-3): Electric analogy model.....                                                                                                      | 20 |
| Figure (2-4): Sand tank model [ Harr, 1962] .....                                                                                              | 21 |
| Figure (2-5): Hele-Shaw model [ Harr, 1962] .....                                                                                              | 22 |
| Figure (2-6): Method of Fragments [ Harr, 1962].....                                                                                           | 23 |
| Figure (3-1): The variables involved in the problem of one soil layer under the apron .....                                                    | 54 |
| Figure (3-2): The variables involved in the problem of stratified soil under the apron.....                                                    | 54 |
| Figure (3-3): A photograph of the electrolytic tank used to study one layer of soil under the apron.....                                       | 55 |
| Figure (3-4): Schematic diagram for the plan of the electrolytic tank considering one layer of soil under the apron .....                      | 55 |
| Figure (3-5): Plan for the mesh used to take the voltage readings in the case of one layer.....                                                | 56 |
| Figure (3-6): A photograph of the electrolytic tank considering two soil layers under the structure.....                                       | 56 |
| Figure (3-7): Schematic diagram for the plan of the electrolytic tank showing the configuration of the two layers. ....                        | 57 |
| Figure (3-8): Plan for the mesh used to take the voltage readings showing the configuration of the two layers.....                             | 57 |
| Figure (3-9): The D.C. power source with rheostat. ....                                                                                        | 58 |
| Figure (3-10): The digital voltmeter.....                                                                                                      | 58 |
| Figure (3-11): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, No Blanket, No Sheet piles.....         | 59 |
| Figure (3-12): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, No Blanket, US Sheet pile .....         | 60 |
| Figure (3-13): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, No Blanket, DS Sheet pile .....         | 61 |
| Figure (3-14): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, No Blanket, US and DS Sheet piles ..... | 62 |
| Figure (3-15): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 5m Blanket Length, No Sheet piles.....  | 63 |

|                                                                                                                                                                                  |    |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure (3-16): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 5m Blanket Length, US Sheet pile .....                                    | 64 |
| Figure (3-17): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 5m Blanket Length, DS Sheet pile .....                                    | 65 |
| Figure (3-18): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 5m Blanket Length, US and DS Sheet piles .....                            | 66 |
| Figure (3-19): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 10m Blanket Length, No Sheet piles.....                                   | 67 |
| Figure (3-20): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 10m Blanket Length, US Sheet pile .....                                   | 68 |
| Figure (3-21): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 10m Blanket Length, DS Sheet pile .....                                   | 69 |
| Figure (3-22): Comparison between SEEP2D and Electric Analogue for the case of a single layer of soil, 10m Blanket Length, US and DS Sheet piles .....                           | 70 |
| Figure (3-23): The effect of different hydraulic conductivity ratios in the case of stratified soil under the apron on equipotential lines for the cases shown in Table (1)..... | 71 |
| Figure (3-24): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_u/K_L = 1$ , $H = 4.5$ m.....                              | 72 |
| Figure (3-25): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_u/K_L = 23.5$ , $H = 4.5$ m.....                           | 73 |
| Figure (3-26): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_u/K_L = 35$ , $H = 4.5$ m.....                             | 74 |
| Figure (3-27): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_u/K_L = 94$ , $H = 4.5$ m.....                             | 75 |
| Figure (3-28): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_u/K_L = 135$ , $H = 4.5$ m.....                            | 76 |
| Figure (3-29): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_u/K_L = 0.0425$ , $H = 4.5$ m.....                         | 77 |

|                                                                                                                                                                         |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure (3-30): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No Sheet piles, $K_v/K_L = 0.0286$ , $H = 4.5$ m.....                | 78  |
| Figure (3-31): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_v/K_L = 0.0106$ , $H = 4.5$ m.....                | 79  |
| Figure (3-32): Comparison between SEEP2D and Electric Analogue for the case of two layers of soil, No sheet piles, $K_v/K_L = 0.0074$ , $H = 4.5$ m.....                | 80  |
| Figure (4-1): The variables involved in the problem .....                                                                                                               | 93  |
| Figure (4-2): Sample of Seep3D domain for the case of no sheet piles and $B = 10$ m.....                                                                                | 94  |
| Figure (4-3): Sample of the finite element mesh for the case of no sheet piles and $B = 10$ m.....                                                                      | 94  |
| Figure (4-4): Sample of Seep3D boundary conditions for the case of no sheet piles and $B = 10$ m.....                                                                   | 94  |
| Figure (4-5): Sample of Seep3D results, head distribution in the soil under the apron for the case of no sheet piles and $B = 10$ m.....                                | 94  |
| Figure (4-6): Seep3D results in Elevation, Plan, and side view .....                                                                                                    | 95  |
| for the case of no sheet piles and $B = 10$ m.....                                                                                                                      | 95  |
| Figure (4-7): Solution of Seep3D showing head distribution under the apron for the case of $B = 5$ m, considering Impermeable (excluded) side banks.....                | 96  |
| Figure (4-8): Uplift distribution under the apron for the studied widths of the structure (at centerline) and the 2D model $H = 4.5$ m .....                            | 97  |
| Figure (4-9): Uplift distribution under the apron for the studied widths of the structure (at sides) compared to the 2D model, $H = 4.5$ m.....                         | 97  |
| Figure (4-10): Exit gradient distribution across the width of the apron, $H = 4.5$ m.....                                                                               | 98  |
| Figure (4-11): Comparison between exit gradient at centerline with the change in the width of the structure from 3D model and its value from 2D model, $H = 4.5$ m..... | 98  |
| Figure (4-12): The ratio between 3D exit gradient and 2D exit gradient at centerline and sides of apron for different $H/B$ values .....                                | 99  |
| Figure (4-13): Uplift distribution under the apron for the studied widths of the structure (at centerline) and the 2D model, $H = 4.5$ m .....                          | 99  |
| Figure (4-14): Uplift distribution under the apron for the studied widths of the structure (at sides) compared to the 2D model, $H = 4.5$ m.....                        | 100 |
| Figure (4-15): Exit gradient distribution across the width of the apron, $H = 4.5$ m.....                                                                               | 100 |

|                                                                                                                                                |     |
|------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure (4-17): The ratio between 3D exit gradient and 2D exit gradient at centerline and sides of apron for different H/B values, H=4.5 m..... | 101 |
| Figure (4- 18) Different scenarios for heterogeneity of sides, No sheet piles, H=4.5m .....                                                    | 102 |
| Figure (4-19) Uplift under apron at centerline of bed, Scenario (1), No sheet piles, B=5m, H=4.5m .....                                        | 103 |
| Figure (4-20) Uplift under apron at sides of bed, Scenario (1), No sheet piles, B=5m, H=4.5m.....                                              | 103 |
| Figure (4-21) Uplift under apron at centerline of bed, Scenario (1), No sheet piles, B=50m, H=4.5m.....                                        | 104 |
| Figure (4-22) Uplift under apron at sides of bed, Scenario (1), No sheet piles, B=50m, H=4.5m.....                                             | 104 |
| Figure (4-23) Uplift under apron at centerline of bed, Scenario (1), Upstream sheet pile, B=5m, H=4.5m.....                                    | 105 |
| Figure (4-24) Uplift under apron at sides of bed, Scenario (1), Upstream sheet pile, B=5m, H=4.5m.....                                         | 105 |
| Figure (4-25) Uplift under apron at centerline of bed, Scenario (1), Upstream sheet pile, B=50 m, H=4.5m.....                                  | 106 |
| Figure (4-26) Uplift under apron at sides of bed, Scenario (1), Upstream sheet pile, B=50 m, H=4.5m .....                                      | 106 |
| Figure (4-27) Uplift under apron at centerline of bed Scenario (2), No sheet piles, B=5m, H=4.5m .....                                         | 107 |
| Figure (4-28) Uplift under apron at sides of bed, Scenario (2), No sheet piles, B=5m, H=4.5m.....                                              | 107 |
| Figure (4-29) Uplift under apron at centerline of bed, Scenario (2), No sheet piles, B=50m, H=4.5m.....                                        | 108 |
| Figure (4-30) Uplift under apron at sides of bed, Scenario (2), No sheet piles, B=50 m, H=4.5m.....                                            | 108 |
| Figure (4-31) Uplift under apron at centerline of bed, Scenario (2), Upstream sheet pile, B=5 m, H=4.5m.....                                   | 109 |
| Figure (4-32) Uplift under apron at sides of bed, Scenario (2), Upstream sheet pile, B=5 m, H=4.5m.....                                        | 109 |
| Figure (4-33) Uplift under apron at centerline of bed, Scenario (2), Upstream sheet pile, B=50 m, H=4.5m.....                                  | 110 |
| Figure (4-34) Uplift under apron at sides of bed S,scenario (2), Upstream sheet pile, B=50 m, H=4.5m .....                                     | 110 |
| Figure (4-35): Exit gradient distribution across the bed, Scenario (1), No sheet piles, B=5m, H=4.5m .....                                     | 111 |
| Figure (4-36): Exit gradient distribution across the bed, Scenario (1), No sheet piles, B=50m, H=4.5m.....                                     | 111 |
| Figure (4-37): Exit gradient distribution across the bed, Scenario (1), Upstream sheet piles, B=5m, H=4.5m .....                               | 112 |

|                                                                                                                                                                                 |     |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure (4-38): Exit gradient distribution across the bed, Scenario (1), Upstream sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                                            | 112 |
| Figure (4-39): Exit gradient distribution across the bed, Scenario (2), No sheet piles, $B=5\text{m}$ , $H=4.5\text{m}$ .....                                                   | 113 |
| Figure (4-40): Exit gradient distribution across the bed, Scenario (2), No sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                                                  | 113 |
| Figure (4-41): Exit gradient distribution across the bed, Scenario (2), Upstream sheet piles, $B=5\text{m}$ , $H=4.5\text{m}$ .....                                             | 114 |
| Figure (4-42): Exit gradient distribution across the bed, Scenario (2), Upstream sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                                            | 114 |
| Figure (4-43): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (1), No sheet piles, $B=5\text{m}$ , $H=4.5\text{m}$ .....                       | 115 |
| Figure (4-44): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (1), No sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                      | 115 |
| Figure (4-45): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (1), Upstream sheet pile, $B=5\text{m}$ , $H=4.5\text{m}$ .....                  | 116 |
| Figure (4-46): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (1), Upstream sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                | 116 |
| Figure (4-47): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (2), No sheet piles, $B=5\text{m}$ , $H=4.5\text{m}$ .....                       | 117 |
| Figure (4-48): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (2), No sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                      | 117 |
| Figure (4-49): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (2), Upstream sheet piles, $B=5\text{m}$ , $H=4.5\text{m}$ .....                 | 118 |
| Figure (4-50): Exit gradient values at centerline and sides for different $K_s/K_b$ , Scenario (2), Upstream sheet piles, $B=50\text{m}$ , $H=4.5\text{m}$ .....                | 118 |
| Figure (4-51): 3D/2D ratios for the exit gradient for different $K_s/K_b$ values, Scenario (1), No Sheet piles, $H=4.5$ .....                                                   | 119 |
| Figure (4-52): 3D/2D ratios for the exit gradient for different $K_s/K_b$ values, Scenario (1), Upstream Sheet pile, $H=4.5$ .....                                              | 119 |
| Figure (4-53): 3D/2D ratios for the exit gradient for different $K_s/K_b$ values, Scenario (2), No Sheet piles, $H=4.5$ .....                                                   | 120 |
| Figure (4-54): 3D/2D ratios for the exit gradient for different $K_s/K_b$ values, Scenario (2), Upstream Sheet pile, $H=4.5$ .....                                              | 120 |
| Figure (4-55): Exit gradient distribution across the bed, Allowing no flow through the two sides of the downstream canal, No sheet piles, $B=5\text{m}$ , $H=4.5\text{m}$ ..... | 121 |

|                                                                                                                                                                                        |     |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Figure (4-56): Exit gradient distribution across the bed, Allowing<br>no flow through the two sides of the downstream canal, No sheet<br>piles, $B=50\text{m}$ , $H=4.5\text{m}$ ..... | 121 |
| Figure (5-1): Critical head calculated from the proposed model<br>against critical head from the data for dune sand.....                                                               | 130 |
| Figure (5-2): Critical head calculated from the proposed model<br>against critical head from the data for dune sand.....                                                               | 130 |
| Figure (5-3): Critical head calculated from the proposed model<br>against critical head from the data for coarse sand .....                                                            | 131 |

## **List of Tables**

|                                                                                                                                                                                                        |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| Table (2-1): Bligh's Thumb Rules for obtaining $L/H_{crit}$ (E) Sellmeijer (1988).....                                                                                                                 | 7   |
| Table (2-2): The correspondence between seepage and flow of electric current.....                                                                                                                      | 19  |
| Table (3-1): The cases of hydraulic conductivities studied using SEEP2D for the stratified model and their effect on the head at point (e) at a depth $d_e = 30\text{m}$ and the total flow rate. .... | 52  |
| Table (4-1): The ratio between 3D exit gradient and 2D exit gradient at centerline and sides of the apron for different $H/B$ values. ....                                                             | 86  |
| Table (4-2): Uplift distribution under the apron for the studied widths of the structure (at centerline) and the 2D model, $H=4.5\text{ m}$ .....                                                      | 87  |
| Table (4-3): Uplift distribution under the apron for the studied widths of the structure (at sides) compared to the 2D model, $H=4.5\text{ m}$ .....                                                   | 87  |
| Table (4-4): The ratio between 3D exit gradient and 2D exit gradient at centerline and sides of the apron for different $H/B$ values, $H=4.5\text{ m}$ .....                                           | 89  |
| Table (4-5): Uplift distribution under the apron for the studied widths of the structure (at centerline) and the 2D model, $H=4.5\text{ m}$ .....                                                      | 89  |
| Table (4-6): Uplift distribution under the apron for the studied widths of the structure (at sides) compared to the 2D model, $H=4.5\text{ m}$ .....                                                   | 90  |
| Table (5-1) Critical head versus porosity observations for different types of sand, reported by Weijers and Sellmeijer (1993).....                                                                     | 128 |
| Table (5-2) Comparing results of the two models according to error function $R_2$ .....                                                                                                                | 128 |
| Note: $H_{crit}$ represents $i_c$ since the length of experiment is $1\text{m}$ .....                                                                                                                  | 128 |
| Table (5-3): porosity data and values of porosity term ( $N$ ) for the three different types of sand .....                                                                                             | 129 |

## **NOTATIONS**

- $B$  = Width of apron [L]
- $d$  = Particle size [L].
- $d_1$  = Depth of upstream cutoff from its point of intersection with the apron to its toe level [L].
- $d_2$  = Depth of downstream cutoff from its point of intersection with the apron to its toe level [L].
- $d_e$  = Depth of point (e) under the downstream bed [L].
- $d_e/T$  = Relative depth of point (e) [Dimensionless].
- $e$  = Any point located on the critical (exit) section along the whole thickness of pervious stratum (T) under the apron.
- $f$  = Coefficient of friction.
- $g$  = Gravitational acceleration [ $LT^{-2}$ ].
- $h$  = Head difference between upstream and downstream sides of the apron [L].
- $h_e$  = Piezometric head at point (e) [L].
- $h_e/h$  = Average relative piezometric head at point (e) [Dimensionless].
- $H$  = Head acting on the structure [L].
- $H_{crit}$  = Critical head [L].
- $h_f$  = Head loss due to friction [L].