University College for Women
(Arts, Science, and Education)
Department of Mathematics

A STUDY ON CONCOMITANTS OF ORDERED VARIABLES

THESIS
SUBMITTED FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY (Ph.D.) IN SCIENCE
(PURE MATHEMATICS)
BY

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"رَبِّ أَوْزِعْنِي أَنْ أَشْكُرَ نِعْمَتَكَ الَّتِي أَنْعَمْتَ عَلَيَّ وَعَلَى وَالِدَيَّ وَأَنْ أَعْمَلَ صَالِحاً تَرْضَاهُ وَأَصْلِحْ لِي فِي ذُرِّيَّتِي إِنِّي تُبْتُ إِلَيْكَ وَإِنِّي مِنْ الْمُسْلِمِينَ"

Before I begin the acknowledgment,
I must kneel thanking God who inspired me,
took my hand and provided me with power, patience and energy.

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Abstract

ABSTRACT

Sohair Khames Khames Gomma. A study on Concomitants of Ordered Variables. Doctor of Philosophy in Science dissertation of Pure Mathematics, University College for women, Ain Shams University.

In this thesis we are concerned with concomitants of ordered variables arising from some generalized forms of the Farlie-Gumbel-Morgenstern (FGM) family of distributions. We introduce general forms of the extensions of the classical bivariate FGM distribution that are proposed by Huang-Kotz, and by Bairamov, Kotz and Bekci, for any continuous marginal distributions. We study the correlation structure of these general extensions for the exponential marginal distributions and the logistic marginal distributions. Also, we derive the probability density function of the concomitant of the *nth* record value and the joint probability density function of the concomitants of *mth* and *nth*, $m<n$ record values arising from these general extensions. The *kth* moment of the concomitant of the *nth* record and the product moment of concomitants of the *mth* and *nth* record values for $m<n$ are also presented. The results are then applied to the case of the two-parameter exponential marginal distributions and two-parameter logistic marginal distributions. Using concomitants of record values we derive the best linear unbiased estimators of parameters of the marginal distributions. Moreover, two methods for obtaining predictors of concomitants of record values are presented and numerical illustration is performed to highlight the theoretical results obtained. We present a new generalization of the trivariate Farlie-Gumbel-Morgenstern distribution (GTFGM) and obtain the admissible range for the association parameter. Dependence properties for this distribution are also discussed. We obtain the distributions of bivariate concomitants of order statistics and record values from GTFGM distribution. We illustrate the results to the case of the two-parameter exponential marginal distributions and the two-parameter logistic marginal distributions.

Keywords: Concomitants, Order statistics, Record values, Generalized Farlie-Gumbel-Morgenstern family, Best linear unbiased estimator, Best linear unbiased predictor.