بسم الله الرحمن الرحيم

شبكة المعلومـــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

قَسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها علي هذه الأفلام قد اعدت دون آية تغيرات

يجب أن

تحفظ هذه الأفلام بعيداً عن الغبار

في درجة حرارة من 15 – 20 مئوية ورطوبة نسبية من 20-40 %

To be kept away from dust in dry cool place of 15 – 25c and relative humidity 20-40 %

شبكة المعلومات الجامعية

بعض الـوثـــائق

الأصـلية تـالفـــة

بالرسالة صفحات

لم ترد بالأصـــــل

Zagazig University
Benha Branch
Faculty of Science
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MATRIX COMPUTATIONS IN CONTROL THEORY

Thesis Submitted By

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In Partial Fulfillment of the Requirements for the M.Sc. Degree " In Pure Mathematics"

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2001

B o C ٤ V

ACKNOWLEDGEMENTS

I would like to express my thanks and indebtedness to my supervisors **DR. SALAH EL-GENDI**, Professor of Mathematics, Faculty of Science, Assuit University for his encouragement and fruitful supervision .

I would like to express my gratitude to **DR. MOHAMED A. RAMADAN** Associate Prof. of Mathematics , Faculty of Science ,El-Menoufia University, who suggested the point of research , for his supervision and guidance during the progress of this work .

Special thanks are due to **DR. MOSTAFA ABD EL-HAMID**, Lecturer, Dept. of Mathematics ,Faculty of Science ,Benha Branch Zagazig University for his valuable help.

I would like to express my gratitude to **DR. MOHAMED SALAH** Lecturer, Dept. of Mathematics , Faculty of Science , Benha Branch Zagazig University, for his valuable help and continuous support .

I am also indebted to all the staff members of the department of mathematics , Faculty of Science , Benha Branch Zagazig University for their support during the preparation of this thesis.

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PREFACE

PREFACE

The matrix exponential plays a central role in linear systems and control theory. Mathematical models of many physical, biological, and economic processes involve systems of linear, constant coefficient ordinary differential equations $\dot{x}(t) = A\,x(t)$ where A is a given fixed, real or complex $n \times n$ matrix. The solution to this equation is given by

$x(t) = e^{At}x(0)$, where $e^{At} = \sum_{k=0}^{\infty} \frac{(At)^k}{k!}$ denotes the exponential of the matrix At and $x(0)$ is the initial solution. In this thesis :

***We** give explicit formulas for computing the exponential of some special matrices and of some special block matrices.

***Also** , we present the use and modification of a recent technique , so called restrictive Padé approximation , used to approximate the exponential matrix and compute the transient response vectors for the above system .

***Also ,** we consider the nonlinear matrix equation

$$X - A^* F(X) A = I \quad , \qquad F(X) = \sqrt{X^{-1}} \,.$$

This type of matrix equation often arises in the analysis of ladder, networks, optimal control theory, dynamic programming, stochastic filtering, and statistics. The iteration process $X_{k+1} = I + A^* F(X_k) A$, $k = 0,1,2,\ldots$ is investigated (under some conditions) to obtain the positive definite solution for this equation . Some numerical examples which describe the performance of the algorithm are presented .

This thesis consists of five chapters where the first two chapters begin with an introduction, in which are given some information and ideas on its contents .

Chapter one :

We summarize some basic concepts, definitions and theorems, for computing the exponential matrix e^A .

Chapter two :

In this chapter we will introduce the linear and nonlinear matrix equations, we present the Lyapunov matrix equation as an example to the linear matrix equation . Also , some properties of the positive definite solution to the nonlinear matrix equation $X+A^T X^{-1} A = I$ are discussed , where the smallest and largest positive definite solutions are mentioned.

Chapter three :

In this chapter, we introduce some explicit formulas for some special types of block matrices that appear very often in control theory. Such block matrices are related to the second-order mechanical vibration equation $M\ddot{x} + C\dot{x} + Kx = 0$ where M, C and K are real or complex $n \times n$ matrices.

Chapter four :

In this chapter, a hybrid technique is presented to compute the transient response vectors in such an iterative explicit form . A recent method , so called restrictive Padé approximation , is used and extended to approximate the exponential matrix [14] . Numerical test example is given to illustrate the accuracy of the suggested method compared with previous works as well as the exact solution.

The computations were done using MATLAB package version 5.1 where long E format were used.

Chapter five :

In this chapter, we establish and prove theorems for the existence of the iteration solution of the nonlinear matrix equation $X - A^{*}\sqrt{X^{-1}}A = I$, $A, X \in P(n)$ where $P(n)$ denotes the set of all positive definite $n \times n$ matrices. Also we obtain the rate of convergence for the sequence . Some numerical examples are given to illustrate the performance of the algorithm . The algorithm is programmed using (Turbo C++ version 3.0) .

CHAPTER (1)

BASIC CONCEPTS OF NUMERICAL LINEAR ALGEBRA AND MATRIX COMPUTATION