

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ

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# **Numerical Solutions for Some Boundary Value Problems of the Electrically Conducting Fluids Flow through Porous Medium with Heat and Mass Transfer**

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3. Numerical treatment of the problem of the MHD fluid flow with heat and mass transfer over a vertical shrinking surface, is accepted for publication in International Journal of Engineering Research and Applications(IJERA).
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5. Numerical solution of MHD stagnation point flow of the Casson fluid with heat and mass transfer over a stretching surface with slip effect, is accepted for publication in Journal of Asian Transactions on Basic & Applied Sciences.
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# Summary

The purpose of this thesis is to investigate the numerical solutions for some boundary value problems of the electrically conducting fluids flow through porous medium with heat and mass transfer. The numerical and analytical solutions have achieved with the aid of MATLAB and MATHEMATICA programmes and all figures are drawn by Excel program. The thesis consists of seven chapters, abstract of the main results in both Arabic and English languages, and the list of references.

## Chapter(1)

In this chapter we concentrated on general introduction about some basics concepts of numerical analysis and fluid mechanics, a brief survey of famous numerical and analytical methods which using to solve some problems of fluid mechanics, such as finite difference method(FDM), differential transform method (DTM) and Multi-step differential transform method (MDTM).

Also, this introduction contains the classifications of the fluids, the motion of the fluid through porous medium, definitions of heat and mass transfer and the effect of external magnetic field on the fluid motion through boundary layer.

## Chapter(2)

The main aim of this chapter is to study the flow of viscous fluid with heat and mass transfer through porous medium past an infinite permeable vertical plate. This system is stressed by a uniform magnetic field in the presence of heat generation and chemical reaction. The governing equations of motion are solved numerically by using (FDM) and analytically by using (DTM). The expressions for the velocity, tem-

perature and concentration have been obtained. The effects of various parameters on these solutions are been computed and discussed in detail through some figures. Comparisons with the analytical solution by using (MDTM) performed and showed that the two solutions are in a good agreement. Finally, the conclusion is summarized.

Some results of this chapter is accepted for:

**International Journal of Pure and Applied Mathematics (IJ-PAM).**

### Chapter(3)

The goal of this chapter is to introduce the comparison between numerical and analytical solution for the motion of viscous fluid with heat and mass transfer through porous medium over a vertical infinite permeable plate in the presence of induced magnetic field. The system is stressed by an external magnetic field, the induced magnetic filed is considered. The viscous dissipation and heat generation are taken in consideration. The system of non-linear coupled equation which arises from momentum, energy, concentration and Maxwell's equations are solved numerically by using finite difference method(FDM) and analytically by using differential transform method (DTM). The solutions are obtained as a functions of the physical problem parameters, then the effects of these parameters on these solutions are illustrated numerically and graphically. Furthermore, comparisons of the numerical results with the analytical results are performed and showed that the two solutions are in a good agreement. Finally, the conclusion is summarized.

Some results of this chapter is published in:

**International Journal of Current Engineering and Technology, 5 1890 –1897 (2015).**

### Chapter(4)

In this chapter, we studied the numerical treatment of the problem of the magnetohydrodynamic fluid flow with heat and mass transfer over a vertical shrinking surface. The mass transfer, radiation and a vertical shrinking surface are taken in consideration. The governing partial differential equations are transformed into nonlinear ordinary

differential equations by applying the similarity transformation and solved numerically by using finite difference method. The effects of various governing parameters, namely, shrinking parameter, magnetic parameter, porosity parameter, radiation parameter, Prandtl number, Schmidt number and chemical reaction parameter on the velocity, temperature and concentration are displayed through some graphs and discussed numerically, as well as the skin friction coefficient, local Nusselt number and Sherwood number. Furthermore, we have compared these results with analytical solution by using differential transform method (DTM). It is observed that this approximate numerical solution is in good agreement with analytical solution. Finally, the conclusion is summarized.

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## **Chapter(5)**

In this chapter, we studied the numerical solution of problem of MHD micropolar fluid flow with heat and mass transfer towards a stagnation point on a vertical plate. The governing equations have been transformed into nonlinear ordinary differential equations by applying the similarity transformation and have been solved numerically by using the finite difference method (FDM) and analytically by using (DTM). The effects of various governing parameters, namely, material parameter, radiation parameter, magnetic parameter, Prandtl number, Schmidt number, chemical reaction parameter and Soret number on the velocity, microrotation, temperature and concentration have been computed and discussed in detail through some figures and tables. Furthermore, the comparisons between numerical and analytical solutions is made. It is observed that this approximate numerical solution is in good agreement with the analytical solution. Finally, the conclusion is summarized.

Some results of this chapter is published in:

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### Chapter(6)

In this chapter, we studied the numerical solution of MHD stagnation point flow of non-Newtonian Casson fluid with heat and mass transfer over a stretching surface with slip effect. The effects of thermal diffusion, diffusion thermo, radiation and Chemical reaction are taken in consideration. The governing partial differential equations are transformed into nonlinear ordinary differential equations by applying the similarity transformation and solved numerically by using finite difference method (FDM). The effects of various governing parameters, on the velocity, temperature and concentration are displayed through graphs and discussed numerically. Furthermore, we have compared these results with analytical solutions by using differential transform method (DTM). It is observed that this approximate numerical solution is in good agreement with analytical solution. Finally, the conclusion is summarized.

Some results of this chapter is accepted for:

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### Chapter(7)

The numerical solution of MHD boundary layer flow of non-Newtonian Casson fluid on a moving wedge with heat and mass transfer and induced magnetic field is studied in this chapter. The effects of thermal diffusion and diffusion thermo with induced magnetic field are taken in consideration. The governing partial differential equations are transformed into nonlinear ordinary differential equations by applying the similarity transformation and solved numerically by using finite difference method (FDM). The effects of various governing parameters, on the velocity, induced magnetic field, temperature and concentration are displayed through graphs and discussed numerically. Furthermore, comparisons of the numerical results with the analytical results are performed and showed that the two solutions are in a good agreement. Finally, the conclusion is summarized.

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